PROSE Heat and Mass Transfer Journal, 2023, Issue 000003

Analytical Modeling of Atmospheric Solar Absorption

Dr. Kevin Woods

Introduction

Modeling atmospheric solar absorptivity analytically is a difficult task using basic modeling software. The general engineer or scientist does not model the absorption spectrum by the atmosphere during their education and are reliant on the results provided by black box computer models that are difficult to validate. This article develops a simplified method for analytically modeling the atmospheric solar absorption spectrum to evaluate the effect of various absorbing substances in the atmosphere on the absorption of sunlight traveling through the atmosphere. The method enables the average engineer or scientist to develop modeling software in a general undergraduate science class using software such as Microsoft Excel™.

Sunlight incident on the atmosphere interacts with gases and clouds through scattering and absorption mechanisms. This analysis considers the atmosphere without clouds and evaluates the overall atmospheric solar absorption of sunlight. The solar absorption is the fraction of solar energy absorbed by the atmosphere. The sun traverses from the horizon at dawn, to a zenith at midday, and back to the horizon at dusk. The angle of incidence of sunlight hitting a flat surface depends on time of day and concentration of gases in the atmosphere. This article incorporates the atmospheric absorption modeling developed in Woods [1] and is aimed for use with the climate model developed by Woods [2]. The output of this article is an annually averaged solar absorption value based on latitude.

This article begins with the general absorption equation developed by Woods [1] with absorption line data from the HITRAN database (www.hitran.org), then evaluates the angle of incidence of sunlight over time for various latitudes to determine the overall solar absorption throughout the year with latitude for a clear sky. The model is built in Microsoft Excel™ and the results are provided for the absorption spectrum of the atmosphere. The atmospheric components included are water vapor, carbon dioxide, ozone, oxygen, and methane, and the solar absorption by each component is separated. Finally, a profile is presented for the overall solar absorption by the atmosphere at varying surface temperatures and latitudes.

Mathematical Model

The portion of solar radiation absorbed by the atmosphere for a specific wavenumber over all absorption lines is approximated in Woods [1] by Eq. (1).

$$a_{S,\nu} = 1 - \frac{I_\nu(0)}{I_\nu(\infty)} = 1 - \prod_i \left(\frac{c_1}{1+c_1}\right)^{c_2} \prod_j \left(\left(1+\frac{1}{b_1}\right)^{b_1} e^{-1}\right)^{b_2} \tag{1}$$

Here a_S is solar absorption at wavenumber ν, and I is radiation intensity. The parameters $c_1 = (\nu - \nu_i)^2/{\gamma_{air,i}}^2{p_o}^2$ and $b_1 = (\nu - \nu_j)^2/{\gamma_{air,j}}^2{p_o}^2$ where the subscript *i* denotes non-condensable substance absorption lines and *j* denotes condensable substance absorption lines (namely water vapor), p_o is the pressure at the surface, and γ_{air} is the absorption half-width of the absorption line by air. The only changes from the atmospheric heat absorption equation in Woods [1] to the atmospheric solar absorption in this article are the parameters c_2 and b_2. Heat absorption and emission were diffuse while solar absorption is highly directional. Therefore, the correction factor of '2' in Woods [1] is replaced with the distance traversed through the atmosphere by a solar photon compared to the distance from the vertical equal to $1/\cos(\theta)$ where θ is the incidence angle of sunlight hitting the surface. Therefore, the parameters are

$$c_2 = \frac{N_A S_i m_s}{2\pi g \gamma_{air,i} M_a \cos(\theta)} \tag{2}$$

$$b_2 = \frac{N_A S_j m_{s,o}}{2\pi g \gamma_{air,j} M_a \cos(\theta)} \tag{3}$$

It should be noted that

$$\lim_{c_1 \to \infty} \frac{c_1}{1 + c_1} = 1 \tag{4}$$

and

$$\lim_{b_1 \to \infty} \left(1 + \frac{1}{b_1}\right)^{b_1} e^{-1} = 1 \tag{5}$$

When the exponent $b_1 > \sim 10^6$, it exceeds the ability for standard spreadsheet programs to calculate accurately. Therefore, for computational purposes, a simplified version of the expression for $b_1 > \sim 1{,}000$ is

$$\left(1 + \frac{1}{b_1}\right)^{b_1} e^{-1} \approx 1 - \frac{1}{2b_1} \tag{6}$$

For $b_1 < 1{,}000$, the actual equation suffices.

Solar Radiation Incidence Angle

Sunlight traverses the sky from dawn to dusk at varying angles from the normal of a flat surface and depends on the day of the year, n, and latitude, l. The declination angle, δ is the angle between the sunlight direction and the equator. The equation for declination angle is

$$\delta = -23.45^\circ \cos\left(2\pi\left(\frac{n}{365}\right)\right) \tag{7}$$

It should be noted that the exact declination angle deviates from the equation with time due to slight changes in the Earth's orbit. However, the general equation is sufficient for the purposes of this article. The incidence angle of sunlight hitting the surface, θ, is equal to the solar zenith angle and given by the following equation.

$$\cos(\theta) = \sin(l)\sin(\delta) + \cos(l)\cos(\delta)\cos(h) \tag{8}$$

Here h is the hour angle where the time the sun traverses past its zenith is 0 hours. Each hour the sun traverses 15° farther from the zenith point. The path length (s) traversed through the atmosphere by a photon compared to the distance traversed by a photon normal to the surface (z) is $s = z/\cos(\theta)$. The only caveat is for angles near 90° the curvature of the Earth inhibits the path length from approaching infinity which would lead to 100% absorption. Instead, the maximum value of $1/\cos(\theta)$ is limited to 50 in the model.

Absorption Spectrum

The absorption lines of substances in the atmosphere are available from the high-resolution transmission molecular absorption database (HITRAN, www.hitran.org). Nitrogen and argon minimally absorb electromagnetic solar radiation but play a role in Rayleigh scattering (value is assumed in this article). Nitrogen composes about 75-80% of the atmosphere, oxygen composes about

18-20% of the atmosphere, argon composes 1% of the atmosphere, water vapor composes between 0-4% of the atmosphere, and trace substances (e.g. carbon dioxide and methane) compose <1% of the atmosphere.

Around 1,000,000 absorption lines were analyzed for water vapor, carbon dioxide, ozone, oxygen, and methane between 0.2 μm and 10 μm wavelengths covering >99% of a sunlight Planck distribution for 5,780 K. The absorption line data were obtained from HITRAN using the highest percentage isotope for each substance. The number of absorption lines were reduced to less than 10,000 by rank ordering from highest to lowest value of $S_j m_{s,o}/\gamma_{air,j}$ from the absorption parameters in Eq. (2) and Eq. (3). This parameter ranges 15 orders of magnitude with the 10,000 highest values covering the highest 6 orders of magnitude. Most absorption lines are water vapor. The strongest ~10 to ~100 absorption lines were included for each substance even if out of the higher orders of magnitude.

Solar radiation differs from heat radiation in that it contains ionizing radiation in the ultraviolet band. This ionizing radiation causes oxygen to convert to ozone in the stratosphere generating the ozone layer. The HITRAN database does not include the ionizing radiation characteristics. However, it is assumed that the UV-C and UV-B bands ranging up to 315 nm are 100% absorbed. This constitutes 4.2% of the energy in the total 5,780 K Planck distribution.

Figure 1 shows the absorption spectrum for sunlight with an angle of incidence of 0° on the surface and an assumed surface temperature of 287 K equal to the actual measured surface temperature of the Earth. The methane concentration assumed is 2 ppm, ozone is 1 ppm, carbon dioxide is 280 ppm, oxygen is 200,000, and water vapor is 15,750 ppm at the 287 K surface.

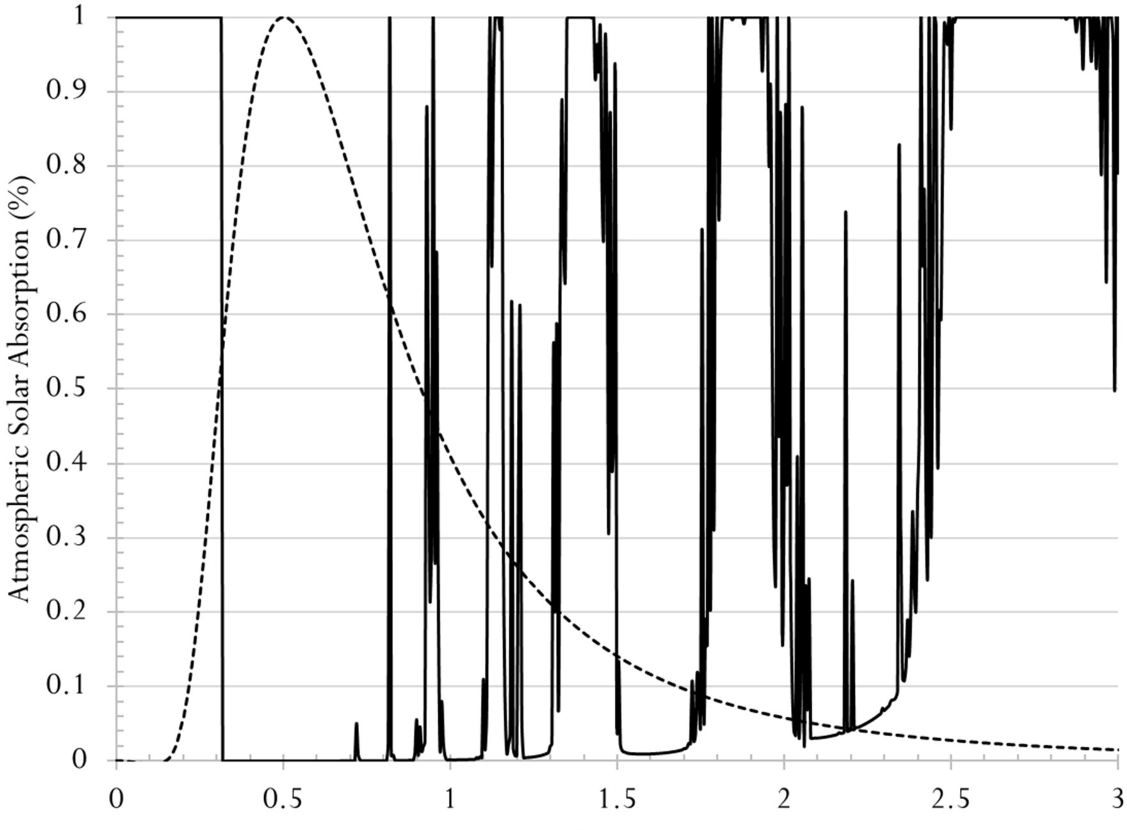
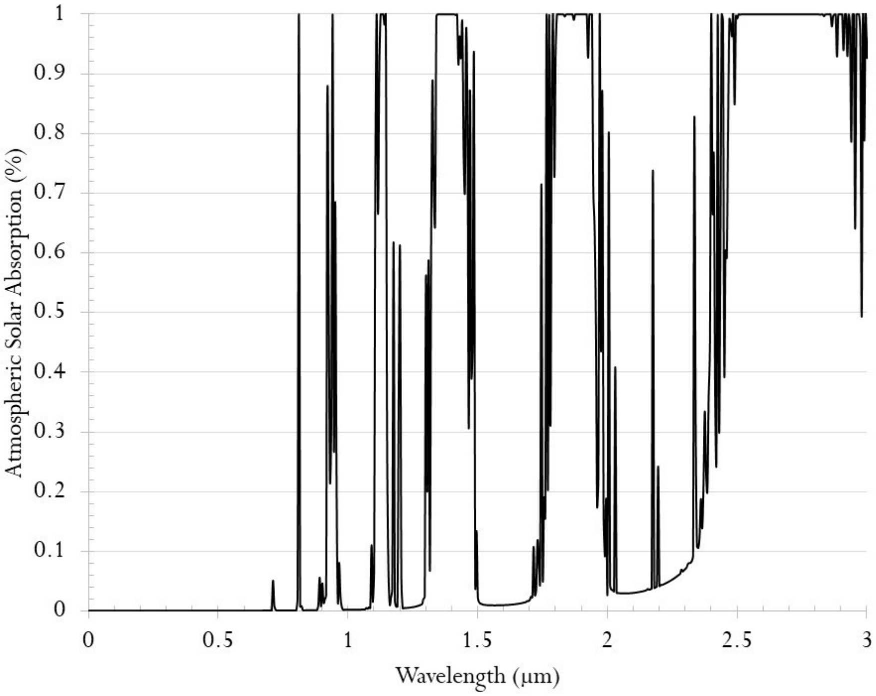


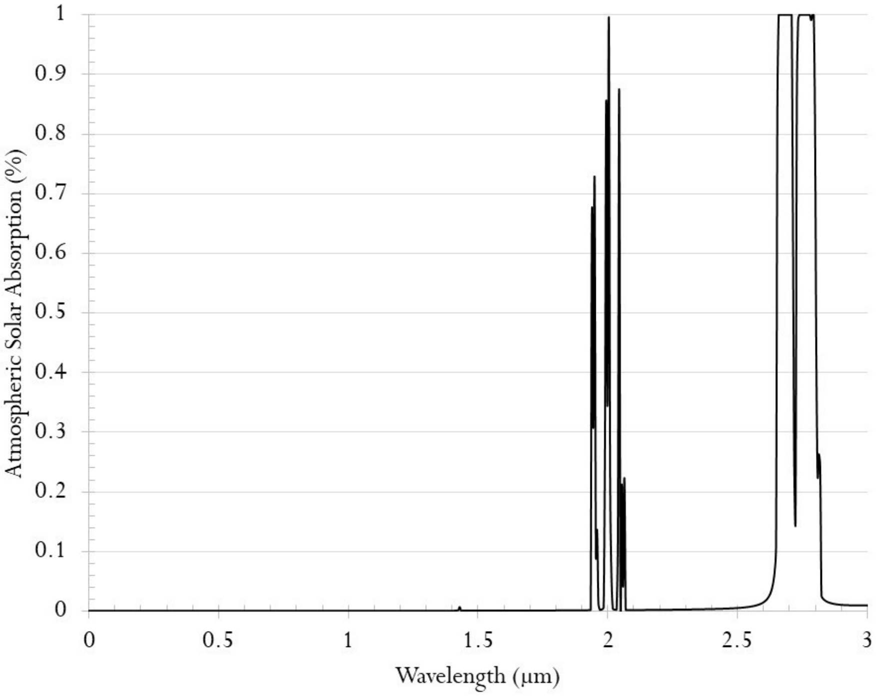
Figure 1: Atmospheric solar absorption spectrum compared with the Planck distribution (dashed line) at 5,780 K simulating solar radiation. Methane is 2 ppm, ozone is 1 ppm, carbon dioxide is 280 ppm, oxygen is 200,000 ppm, and water vapor is 15,750 ppm from 287 K.

The range between ~300 nm and ~800 nm is a solar atmospheric window that coincides with the peak of the 5,780 K Planck distribution. The range from 400 nm to 700 nm is visible light.

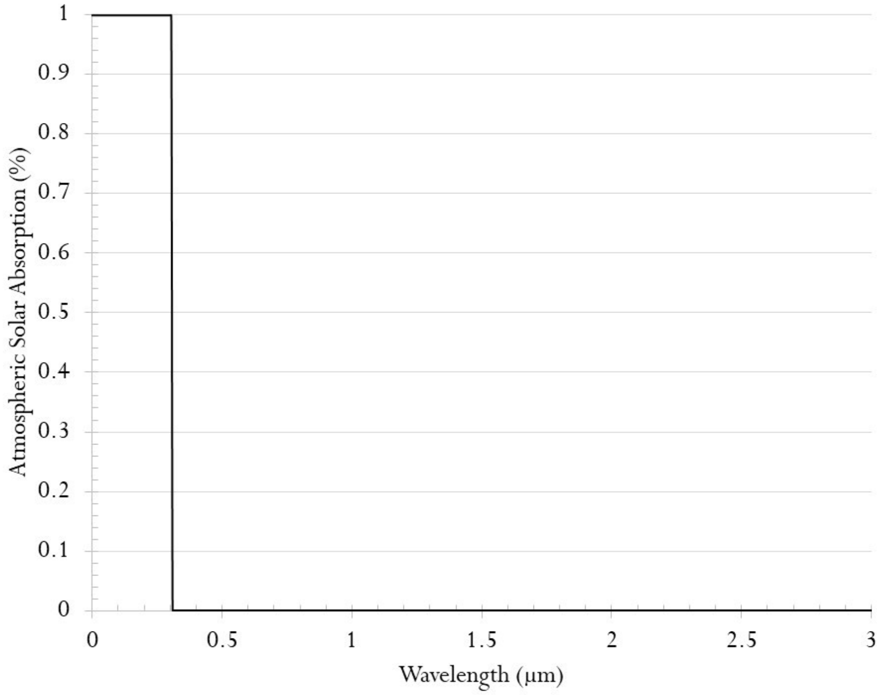
Figure 2 shows the breakdown of the absorption spectrums based on individual atmospheric constituents. Methane absorption is negligible.



(a)



(b)



(c)

Figure 2: (a) Solar absorption spectrum of water vapor. (b) Solar absorption spectrum of carbon dioxide. (c) Solar absorption spectrum of ozone and oxygen.

Not included in this analysis is the Rayleigh scattering of sunlight by the atmosphere producing the blue sky. This scattering leads to backscattering (reflection) of about 5% of sunlight into space. Rayleigh scattering scatters about 50% of UV-A photons (315 nm – 400 nm) reducing to about 25% of blue light (400 nm) and <5% of red light (700 nm).

The main source of absorption is water vapor in Figure 2(a). Water vapor concentration changes depending on the surface temperature of the Earth with warmer climates having higher water vapor concentrations and cooler climates having lower concentrations. Interestingly, the photons traversing the atmosphere in warmer climates with higher water vapor concentration have lower path lengths, and vice versa for cooler climates. The next section will delve into this dichotomy.

Atmospheric Solar Absorption Model Results

Figure 3 is produced through integration of the absorption model and variation of angle of incidence and surface temperature. The surface temperature affects the atmospheric water vapor concentration. Figure 3 is the atmospheric solar absorptivity. The atmospheric solar absorption is the absorptivity weighted against the relative intensity of sunlight hitting a surface at a given latitude. Another way to consider the atmospheric solar absorption is the total percent of energy absorbed by the atmosphere that would have hit a flat surface without the atmosphere present. Figure 3 shows that the influence of water vapor on solar absorption is significant. At 220 K, the water vapor concentration is ~10 ppm while at 320 K the concentration is about ~100,000 ppm.

The absorptivity model is applied for every hour of the year at a given latitude and assumed surface temperature with a solar incidence angle determined from Eq. (8). The absorption for each hour is applied to the solar energy incident on a flat surface at the given latitude for the hour and then averaged over a year. The solar energy equation is

$$q_S = S\cos(\theta) \tag{9}$$

Here S is the solar constant of 1365 W/m^2 and q_S is the solar flux incident on a flat surface without atmospheric effects. At an incidence angle of 90°, the solar energy incident on a flat surface is 0 W/m^2 while at an incidence angle of 0° the solar energy incident equals S.

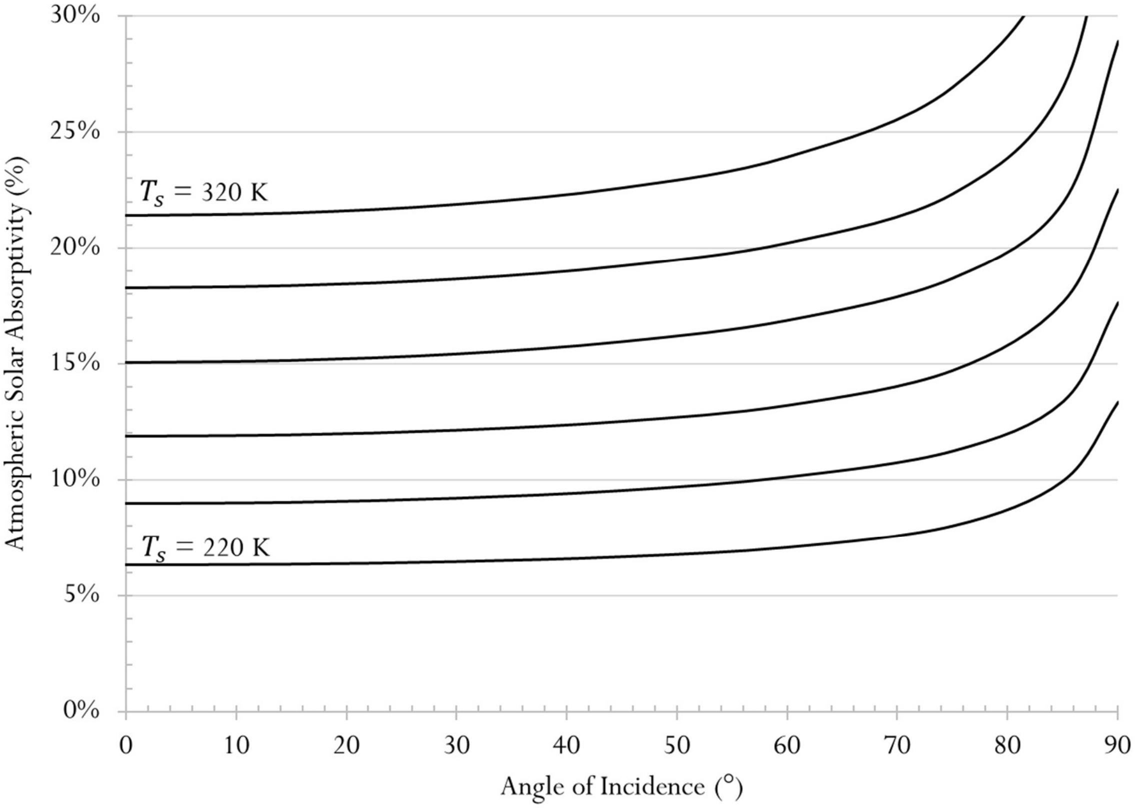


Figure 3: Atmospheric solar absorptivity variation with surface temperature and angle of incidence through the atmosphere

At 90° the atmospheric solar absorption (a_S) is 0% because it is the solar energy incident on the flat surface (0 W/m^2) times the atmospheric solar absorptivity (α_S) at 90°. For angles greater than 90°, the solar energy is 0 W/m^2. The results for atmospheric solar absorption are shown in Figure 4. The average surface temperature in the equatorial zone (30°S to 30°N) is ~300 K. The average temperature in the temperate zones (30° to 60°N and S) is between ~280 and ~300 K. The polar regions (60° to 90°N and S) range from ~250 K to ~270 K.

Figure 4 suggests that the overall atmospheric solar absorption does not vary significantly with latitude as equatorial regions experience $a_S \approx$ 19-20%, temperature regions $a_S \approx$ 17-18%, and polar regions $a_S \approx$ 15-16%. The article Woods [2] assumed an atmospheric solar absorption of $a_S = 15\% \pm 5\%$ capturing this variation. A closer approximation for the average solar absorption over the Earth is $a_S \approx 18\% \pm 1\%$.

Using Figure 4 and temperature data from the CRU database [3], an equation of absorption with latitude (l) is produced in Eq. (10).

$$a_S = 0.20 - 0.05\sin(l) \tag{10}$$

There is minimal variation in solar absorption across the Earth. When Rayleigh scattering (R_r) is integrated, the total absorption by the atmosphere plus backscattering to space is: $a_S + R_r = 23\% \pm 2\%$.

A more robust equation for the solar absorption with latitude varies with temperature and is provided in Eq. (11). This equation is more adept to an analysis that delves into ice age temperature swings.

$$a_S = 0.125 + 0.015\left(1 + erf\left(\frac{l - 60°}{15°}\right)\right) + \left(0.0017 + 0.00045\left(\frac{l}{90°}\right)\right)(T_s - 260K) \tag{11}$$

The variation utilizes the error function to capture the transition around 60° latitude.

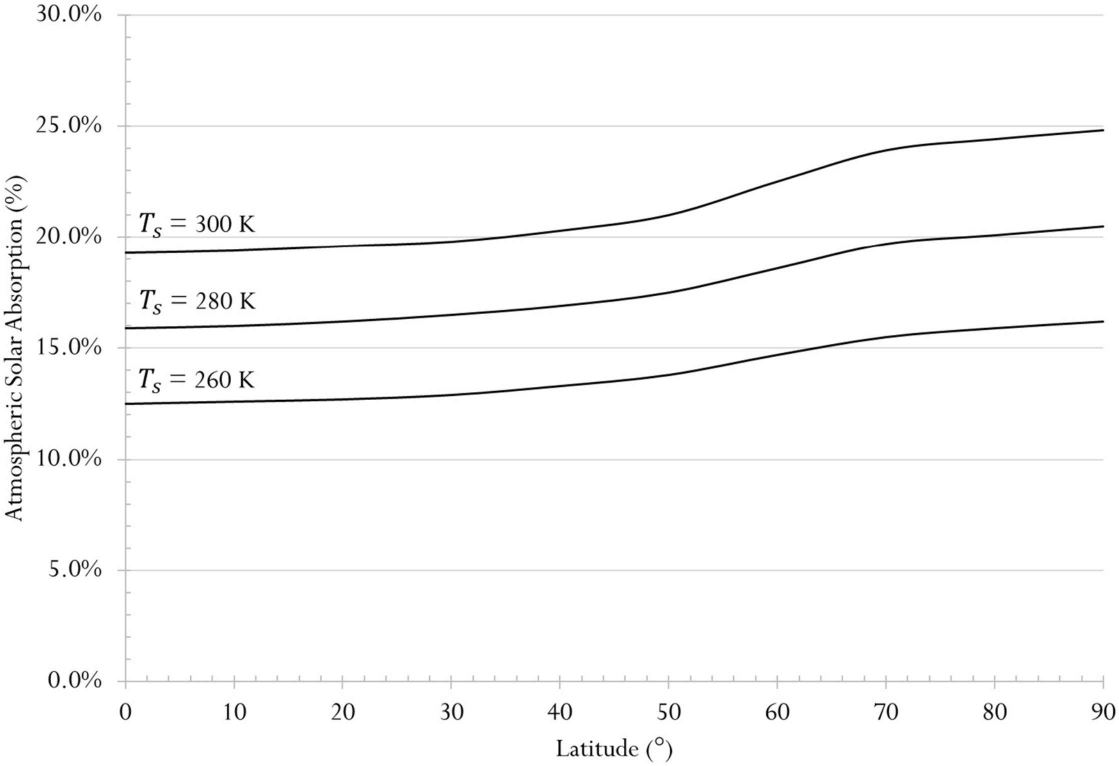


Figure 4: Percent of sunlight reaching the surface that is absorbed at a given latitude and surface temperature

Conclusion

The atmospheric solar absorption spectrum requires solution of non-linear equations using significant computation power. This article developed an analytical equation for modeling the absorption of radiation for a given wavenumber as the radiation traverses the atmosphere. The model integrates an approximate model from Woods [1] utilized previously for atmospheric heat absorption to determine atmospheric solar absorption. The absorption spectrum was developed in Microsoft Excel™ using absorption line data from HITRAN (hitran.org) by only including absorption lines producing significant absorption of radiation.

Applying the absorption spectrum to a Planck distribution of solar energy from the sun deduced a profile for the overall atmospheric solar absorption at differing surface temperatures. The variation of the model with latitude was determined showing a general increase in solar absorption with latitude at a constant temperature. The conclusion was the atmospheric solar absorption is $a_S \approx 18\% \pm 1\%$ across the Earth with polar regions at the lower end of the range and equatorial regions at the higher end of the range.

Acknowledgements

The author would like to acknowledge and thank the high-resolution transmission molecular absorption database (HITRAN.org) for their open access absorption line data utilized in the analysis. HITRAN was developed and is maintained at the Atomic and Molecular Physics Division of the Harvard-Smithsonian Center for Astrophysics.

Appendices

There are no appendices for this research.

References

[1] K. Woods, "Analytical Modeling of Atmospheric Heat Absorption and Emission," *PROSE Heat and Mass Transfer Journal,* no. 000002, 2023.

[2] K. Woods, "Modeling Climatic Temperature Using The Stability Solution," *PROSE Journal of Thermal Science,* no. 000001, 2023.

[3] Climate Research Unit, "High Resolution Gridded Datasets," University of East Anglia, May 2023. [Online]. Available: https://crudata.uea.ac.uk/cru/data/hrg/#current. [Accessed 13 October 2023].

Author Contact Information

Email: kevin.woods8@gmail.com, woodsclimatemodel@gmail.com
Info: Editor at PROSE Journals / Principal Mechanical Engineer at Honeywell Inc / Adjunct Faculty at Villanova University