Analytical Modeling of Atmospheric Heat Absorption and Emission

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Introduction

Modeling atmospheric heat absorptivity is a difficult task using basic modeling software. The general engineer or scientist does not model the absorption spectrum by the atmosphere during their education and are reliant on the results provided by black box computer models in the literature. This article develops a simplified method for modeling the atmospheric absorption spectrum to evaluate the effect of various absorbing substances in the atmosphere on the absorptivity and emissivity. The method enables the average engineer or scientist to develop modeling software in a general undergraduate science class using software such as Microsoft Excel^{TM}.

Atmospheric heat absorptivity and emissivity is approximated in Incropera and Dewitt [1] through a combination of the emissivity of water vapor and carbon dioxide. Utilizing the graphs from p. 823-826 in [1], the gas emissivity is the addition of the emissivity of water vapor equal to ~0.7 at 300 K plus the emissivity of carbon dioxide equal to ~0.2 at 300 K minus a correction factor equal to ~0.05 at 300 K; totaling ~0.85. Evaluating the variation of this absorption with temperature is difficult with the given data. The atmospheric heat absorption results are complementary to a climate model developed by the author in [2] and expand the Stability Solution model in [2] from five (5) equations to six (6) equations.

This article begins with the general Lorenz absorption distribution neglecting doppler absorption, removes the effect of pressure shift on the absorption profile for each absorption line, and removes the effect of self-broadening to yield a Lorenz distribution that can be integrated. The Lorenz profile is solely due to the absorption line parameters and pressure broadening of the line. The absorption profiles for the absorption lines are integrated together using superposition. The absorption of radiation through the atmosphere is determined through integration with altitude using a pressure gradient derived from first principles with an assumed average atmospheric temperature. The solution for overall absorption is an elegant product solution of the absorption lines of the substances in the atmosphere. However, the solution is limited as it does not integrate variation of composition with altitude that occurs with water vapor.

The variation of concentration with altitude for water vapor is integrated into the problem through use of an approximate solution derived from the Clausius-Clapyron relation. The solution is integrated with altitude again with this additional concentration variation yielding another elegant solution for the overall absorption of radiation by the atmosphere. The solution is a product solution of the absorption lines again but only is valid for radiation emitted normal to the surface.

Diffuse radiation such as the heat emitted by the surface of the Earth is emitted in all directions. The radiation emitted at an angle from the normal traverses a longer distance through the atmosphere and interacts with additional absorbing molecules. The integration of this fact is determined using the average distance traversed through the atmosphere by photons emitted by a flat surface determined to be 2 times the distance traversed normal to the surface. This doubling factor is integrated yielding a final solution for the overall absorption of radiation emitted by Earth's surface and absorbed by Earth's atmosphere.

The model is built in Microsoft Excel^{TM} and the results are provided for the absorption spectrum of the atmosphere. The atmospheric components included are water vapor, carbon dioxide, ozone, and methane, and the heat absorption by each component is separated. Finally, a profile is presented for the overall absorption by the atmosphere for radiation emitted by Earth's surface at varying surface temperatures and with varying concentrations of carbon dioxide.

Mathematical Derivation

A beam of radiation with a specific wavenumber (ν) will be absorbed by a gas depending on the absorptivity of the gas, and the length the radiation traverses. The wavenumber is the inverse of the wavelength. The rate of change of radiation with distance is proportional to the absorptivity times the intensity of the radiation (I_ν) given by

$$\frac{dI_v}{dz} = -I_v \alpha_v(z) \tag{1}$$

The absorption coefficient (α_v in m^{-1}) is a function of the monochromatic absorption coefficient (κ_v) in m^2/mole times the absorbing particle density (ρ) in moles/m^3 yielding

$$\frac{dI_v}{dz} = -I_v \kappa_v(z) \rho(z) \tag{2}$$

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The monochromatic absorption coefficient is a combination of the spectral line intensity (S) in m/molecules of absorbing substance times a Lorentz absorption profile in 1/m^{-1} around the absorbing wavenumber produced by pressure broadening of the absorption line.

$$\kappa_v(z) = \left(\frac{N_A S}{\pi M_a}\right) \frac{\gamma(p,T)}{\gamma(p,T)^2 + [v - (v_i + \delta p)]^2} \tag{3}$$

Here p is the total pressure in atm, T is the temperature in Kelvin, S is the line intensity in m^{-1}/(molecule/m^2), v_i is the absorbing wavenumber in m^{-1}, δ is the pressure shift in m^{-1}/atm of the absorbing wavenumber, N_A is Avogadro's number in molecules/mole, M_a is the molar mass of air, and $\gamma(p,T)$ is the pressure broadening half-width. The pressure broadening half-width is determined using the following equation:

$$\gamma(p,T) = \left(\frac{T_{ref}}{T}\right)^{n_{air}} \left(\gamma_{air}(p_{ref}, T_{ref})(p - p_{self}) + \gamma_{self}(p_{ref}, T_{ref})p_{self}\right) \tag{4}$$

Here $T_{ref} = 296$ K and $p_{ref} = 1$ atm are the reference data temperature and pressure, n_{air} is the coefficient of the temperature dependence of the air-broadening half width, γ_{air} is the air broadening half width at half maximum in m^{-1}/atm at T_{ref} and p_{ref}, and p_{self} is the pressure of the absorbing molecule. Assuming $p_{self} \ll p$, the temperatures examined are near T_{ref}, and $n_{air} \rightarrow 0$, then the pressure broadening half-width simplifies to

$$\gamma(p,T) \approx \gamma_{air} p \tag{5}$$

The density of an ideal gas is $\rho(z) \approx m_s p(z)/RT_{av}$ where m_s is the molar concentration of the absorbing substance in the atmosphere, R is the universal gas constant, and T_{av} is the average temperature of the atmosphere in Kelvin. Assuming the pressure shift $\delta \rightarrow 0$, the absorptivity simplifies to

$$\alpha_v(z) = \frac{N_A S m_s}{\pi \gamma_{air} M_a R T_{av}} \frac{(\gamma_{air} p(z))^2}{(\gamma_{air} p(z))^2 + (v - v_i)^2} \tag{6}$$

The pressure of the atmosphere is determined using a force balance given by

$$p_z A - p_{z+dz} A = \rho g A dz \tag{7}$$

Here A is an area that cancels out, and g is the gravitational constant (~9.8 m/s^2). Using a Taylor-series expansion, the pressure drop in the Earth's gravitational field becomes

$$\frac{dp}{dz} = -\rho g \tag{8}$$

Applying the ideal gas law, and integrating produces

$$\ln(p) = -\frac{gz}{RT_{av}} + C_1 \tag{9}$$

Using the atmospheric pressure at the surface (p_o) of the Earth to solve for C_1, the pressure profile with altitude becomes

$$p(z) = p_o e^{-\frac{gz}{RT_{av}}} \tag{10}$$

Applying this atmospheric pressure profile yields an absorptivity of

$$\alpha_v(z) = \frac{N_A S m_s}{\pi \gamma_{air} M_a RT_{av}} \frac{\gamma_{air}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}}}{\gamma_{air}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}} + (v - v_i)^2} \tag{11}$$

Rearranging Eq. (1) and inputting the absorptivity yields

$$\frac{dI_v}{I_v} = -\frac{N_A S m_s}{\pi \gamma_{air} M_a RT_{av}} \frac{\gamma_{air}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}}}{\gamma_{air}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}} + (v - v_i)^2} dz \tag{12}$$

The absorptivity so far is for one absorption wavenumber. The change in the radiation intensity is a superposition of all absorption wavenumbers modifying Eq. (12) to

$$\frac{dI_v}{I_v} = -\sum_i \left(\frac{N_A S_i m_s}{\pi \gamma_{air,i} M_a RT_{av}} \right) \frac{\gamma_{air,i}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}}}{\gamma_{air,i}^2 p_o^2 \exp\left(-2\frac{gz}{RT_{av}}\right) + (v - v_i)^2} dz \tag{13}$$

Integrating throughout the atmosphere from ground surface to the top of the atmosphere yields

$$\ln(I_v(\infty)) - \ln(I_v(0)) = -\sum_i \frac{N_A S_i m_s}{\pi \gamma_{air,i} M_a RT_{av}} \int_0^\infty \frac{\gamma_{air,i}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}}}{\gamma_{air,i}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}} + (v - v_i)^2} dz \tag{14}$$

The integral is solved by assuming $u = e^{-\frac{2gz}{RT_{av}}}$ and $du = -(2g/RT_{av})e^{-\frac{2gz}{RT_{av}}}dz$ producing

$$\ln\left(\frac{I_v(\infty)}{I_v(0)}\right) = \sum_i \frac{N_A S_i m_s}{\pi \gamma_{air,i} M_a RT_{av}} \frac{RT_{av}}{2g} \int_1^0 \left(\frac{1}{u + c_1}\right) du \tag{15}$$

Here $c_1 = (v - v_i)^2 / \gamma_{air,i}^2 p_o^2$. Assuming $c_2 = N_A S_i m_s / 2\pi g \gamma_{air,i} M_a$ produces

$$\ln\left(\frac{I_v(\infty)}{I_v(0)}\right) = \sum_i c_2 \ln\left(\frac{c_1}{1 + c_1}\right) \tag{16}$$

The radiation intensity traversing through the atmosphere is

$$\frac{I_v(\infty)}{I_v(0)} = e^{\sum_i c_2 ln\left(\frac{c_1}{1+c_1}\right)} \tag{17}$$

The summation of an exponent is the product of exponents given by

$$\frac{I_v(\infty)}{I_v(0)} = \prod_i e^{c_2 ln\left(\frac{c_1}{1+c_1}\right)} \tag{18}$$

The exponent is canceled by the logarithm. Rearranging produces

$$\frac{I_v(\infty)}{I_v(0)} = \prod_i \left(\frac{c_1}{1+c_1}\right)^{c_2} \tag{19}$$

The percent of radiation absorbed by the atmosphere (a_A) at each wavenumber is

$$a_{A,v} = 1 - \frac{I_v(\infty)}{I_v(0)} = 1 - \prod_i \left(\frac{c_1}{1+c_1}\right)^{c_2} \tag{20}$$

Water Vapor Condensation

The water vapor concentration changes significantly in the atmosphere due to condensation. Non-condensing substance concentrations depend on the pressure variation with altitude while condensing substance concentrations vary with temperature with altitude. The portion of absorption lines that are water vapor need to be modified due to the change in the concentration with altitude since the previous analysis overestimates the absorption by water vapor. The change in concentration is determined by the vapor pressure of water. The vapor pressure change of water vapor with altitude is approximated using the Clausius-Clapeyron equation given by

$$\frac{dp_{H_2O}/p_{H_2O}}{dz} = \frac{\Delta h_{fg}}{RT^2}\frac{dT}{dz} = \frac{dln(p_{H_2O})}{dz} = \frac{dln(m_{H_2O})}{dz} \tag{21}$$

The rate of change of temperature (known as the adiabatic lapse rate) in the troposphere is $dT/dz = -\gamma = -6.5$ K/km. The natural logarithm yields a concentration with altitude approximated by

$$m_{H_2O}(z) = m_{s,o}e^{-\beta z} \tag{22}$$

Here $m_{s,o}$ is the molar concentration of water vapor at the Earth's surface determined by the surface temperature. The coefficient β using the average Earth surface temperature and adiabatic lapse rate is approximated by

$$\beta = \frac{\Delta h_{fg}}{R_w T_o^2}\gamma = 0.00043m^{-1} \tag{23}$$

Here Δh_{fg} is the latent heat of condensation of water vapor, R_w is the gas constant for water vapor, and T_o is Earth's average surface temperature of ~287 K. β is about 1.6 times the pressure exponent $2g/RT_{av} = 0.00026$ m⁻¹.

Near the equator where the surface temperature is ~300K and rate of change of temperature is $\gamma \approx 4$ K/km, $\beta \approx 2g/RT_{av}$. Figure 1 shows the assumed variation in concentration versus the variation at the equator, the average Earth temperature, and compared to non-condensing concentration with altitude. For a gas like carbon dioxide, the concentration is 280 ppm at the surface and 280 ppm at 10 km altitude producing a ratio of 1 as depicted for non-condensing substances.
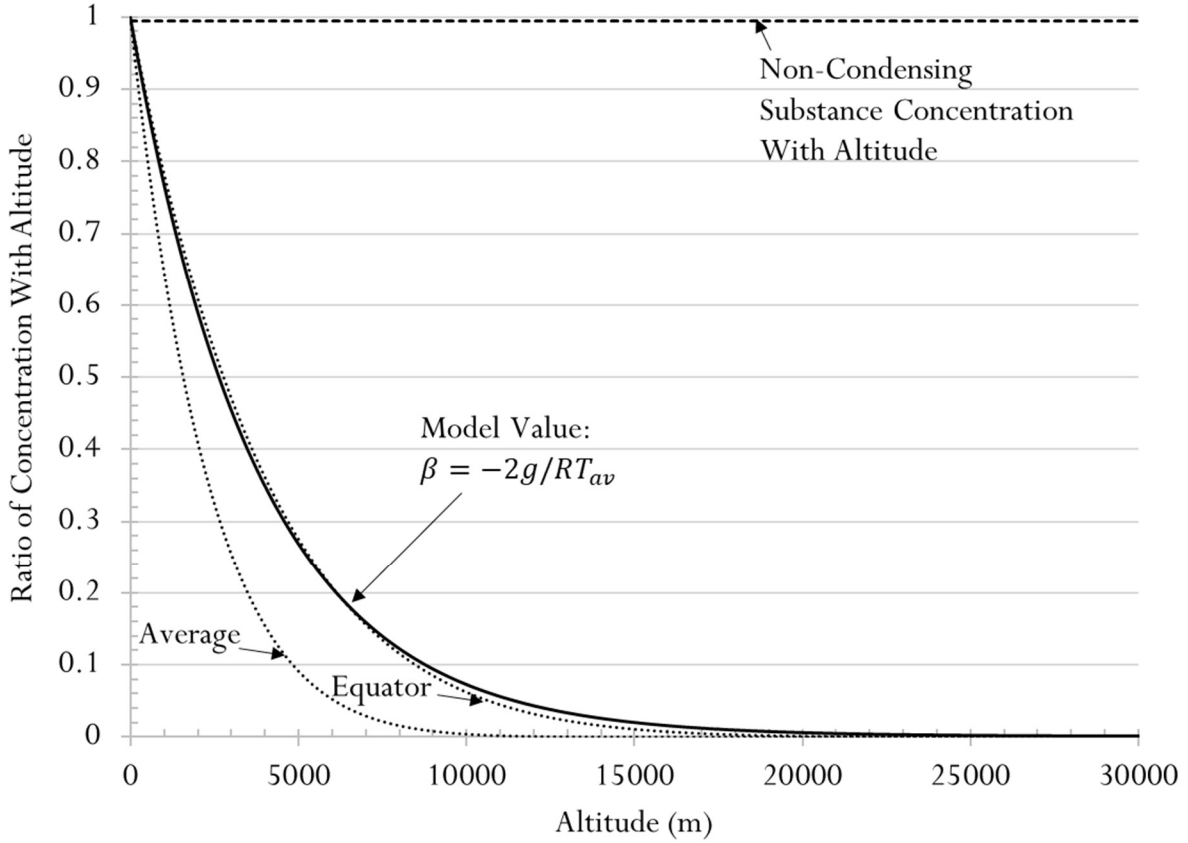
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Figure 1: Ratio of concentration variation with altitude for non-condensing substances versus water vapor condensation at the equator and on average. The solid line is the modeled variation approximating the water vapor variation.

Separating Eq. (13) into water vapor with $\beta \approx 2g/RT_{av}$ and non-condensing substances like carbon dioxide produces Eq. (24).

$$\frac{dI_v}{I_v} = -\sum_i \frac{N_A S_i m_s}{\pi \gamma_{air,i} M_a R T_{av}} \frac{\gamma_{air,i}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}}}{\gamma_{air,i}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}} + (v - v_i)^2} dz$$
$$- \sum_j \frac{N_A S_j m_{s,o}}{\pi \gamma_{air,j} M_a R T_{av}} e^{-\beta z} \frac{\gamma_{air,j}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}}}{\gamma_{air,j}^2 p_o^2 e^{-\frac{2gz}{RT_{av}}} + (v - v_i)^2} dz$$

(24)

For the case of $\beta \approx 2g/RT_{av}$, the integral of the summation in Eq. (24) with condensation is completed by assuming $x = e^{-\lambda z}$ and $dx = -\lambda e^{-\lambda} dz$ where $\lambda = 2g/RT_{av}$, and $b_1 = (v - v_j)^2/\gamma_{air,j}^2 p_o^2$. Applying integration by parts produces

$$-\frac{1}{\lambda} \int_1^0 \frac{x}{x + b_1} dx = -\frac{1}{\lambda}[-b_1 ln(x + b_1) + (x + b_1)]_1^0 = \frac{1}{\lambda}\left[b_1 ln\left(\frac{b_1}{1 + b_1}\right) + 1\right]$$

(25)

The concentration variation with altitude is commensurate with the actual water vapor concentration with altitude but is still an overestimate in the lower atmosphere and becomes an underestimate in the upper atmosphere when condensation ceases. However, the integral improves the analytical model by modifying the absorption to partially compensate for the effect of condensation.

Applying the integral and assuming $b_2 = N_A S_j m_{s,o}/2\pi g \gamma_{air,j} M_a$ yields

$$ln\left(\frac{I_v(\infty)}{I_v(0)}\right) = \sum_i c_2 ln\left(\frac{c_1}{1+c_1}\right) - \sum_j b_2\left[b_1 ln\left(\frac{b_1}{1+b_1}\right) + 1\right] \tag{26}$$

It should be noted that b_2 and c_2 are similar since c_2 utilizes a constant substance concentration that equals the substance concentration at the surface which is used in b_2. Removing the natural logarithm modifies the summation into a product solution in Eq. (27).

$$\frac{I_v(\infty)}{I_v(0)} = \prod_i\left(\frac{c_1}{1+c_1}\right)^{c_2} \prod_j\left(\left(1+\frac{1}{b_1}\right)^{b_1} e^{-1}\right)^{b_2} \tag{27}$$

The portion of radiation absorbed by the atmosphere is for a specific wavelength over all absorption wavenumbers is

$$a_{A,v} = 1 - \frac{I_v(\infty)}{I_v(0)} = 1 - \prod_i\left(\frac{c_1}{1+c_1}\right)^{c_2} \prod_j\left(\left(1+\frac{1}{b_1}\right)^{b_1} e^{-1}\right)^{b_2} \tag{28}$$

Integrating condensation causes the absorption of radiation by the atmosphere to decrease due to the presence of fewer absorbing molecules. It should be noted that

$$\lim_{c_1 \to \infty} \frac{c_1}{1+c_1} = 1 \tag{29}$$

and

$$\lim_{b_1 \to \infty}\left(1+\frac{1}{b_1}\right)^{b_1} e^{-1} = 1 \tag{30}$$

When the exponent $b_1 > {\sim}10^6$, it exceeds the ability for standard spreadsheet programs to calculate accurately. Therefore, for computational purposes, a simplified version of the expression for $b_1 > {\sim}1{,}000$ is

$$\left(1+\frac{1}{b_1}\right)^{b_1} e^{-1} \approx 1 - \frac{1}{2b_1} \tag{31}$$

For $b_1 < 1{,}000$, the actual equation suffices.

Diffuse Heat Emission

The analysis to this point has assumed an emitted radiation solely in the vertical direction. Diffuse radiation is emitted at all angles from a flat surface and must pass through a longer length of atmosphere before reaching space. A photon emitted at 30°, 45°, and 60° from the normal traverses 1.15, 1.41, and 2 times the distance traversed in the atmosphere by a photon emitted at 0° from the normal, respectively. This additional distance is necessary to model the actual overall absorption and is modeled using the view factor.

The view factor from a differential surface to a circular surface located above is

$$F_{1-2} = \frac{\tan^2(\theta)}{1+\tan^2(\theta)} \tag{32}$$

Here θ is the angle from the normal. Subtracting the view factor for θ from the view factor for $\theta + d\theta$ yields the percent of diffuse radiation emitted at a given angle. Multiplying this value by $1/\cos{(\theta)}$ yields the average distance that the diffuse radiation traverses through the atmosphere at the angle. The average distance traversed by all photons is 2 times the distance traversed by a photon emitted at $\theta = 0°$.

The distance the photon traverses (s) is related to the altitude (z) by $s = z/2$. The integration through altitude in the previous sections is replaced with the average distance traversed by a photon and integrated doubling the values for b_2 and c_2.

$$b_2 = c_2 = \frac{N_A S_j m_{s,o}}{\pi g \gamma_{air,j} M_a} \tag{33}$$

Absorption Spectrum

The absorption lines of substances in the atmosphere are available from the high-resolution transmission molecular absorption database (HITRAN). Nitrogen and argon interact minimally with electromagnetic radiation in the range of interest in the infrared characteristic of heat radiation. Nitrogen composes 75-80% of the atmosphere, oxygen composes 18-20% of the atmosphere, argon composes 1% of the atmosphere, water vapor composes between 0-4% of the atmosphere, and trace substances (e.g. carbon dioxide and methane) compose <1% of the atmosphere.

Around 1,000,000 absorption lines were analyzed for water vapor, carbon dioxide, ozone, and methane between 2 μm and 100 μm wavelengths. The absorption line data were obtained from HITRAN using the highest percentage isotope for each substance. The number of absorption lines were reduced to less than 10,000 by rank ordering from highest to lowest value of $S_j m_{s,o}/\gamma_{air,j}$. This value ranges 15 orders of magnitude with the 10,000 highest covering the highest 7 orders of magnitude. Most absorption lines are water vapor. The strongest ~100 to ~1,000 absorption lines were included for each substance even if out of the higher orders of magnitude.

Atmospheric Heat Absorption

The concentration of water vapor is determined using the vapor pressure of water at the surface temperature. The pre-industrial concentration of carbon dioxide is 280 ppm while 1 ppm ozone and 2 ppm methane are assumed. Figure 1 shows the absorption spectrum for Earth's approximate average surface temperature of 287 K. The reduced absorption between ~7 and 14 μm wavelengths is considered the "atmospheric window" and is captured by the analysis. The spike between ~9 and 10 μm is due to ozone absorption mainly in the stratosphere between ~20 and 50 km altitude. All other absorption occurs mainly in the troposphere and tropopause between 0 and ~20 km.
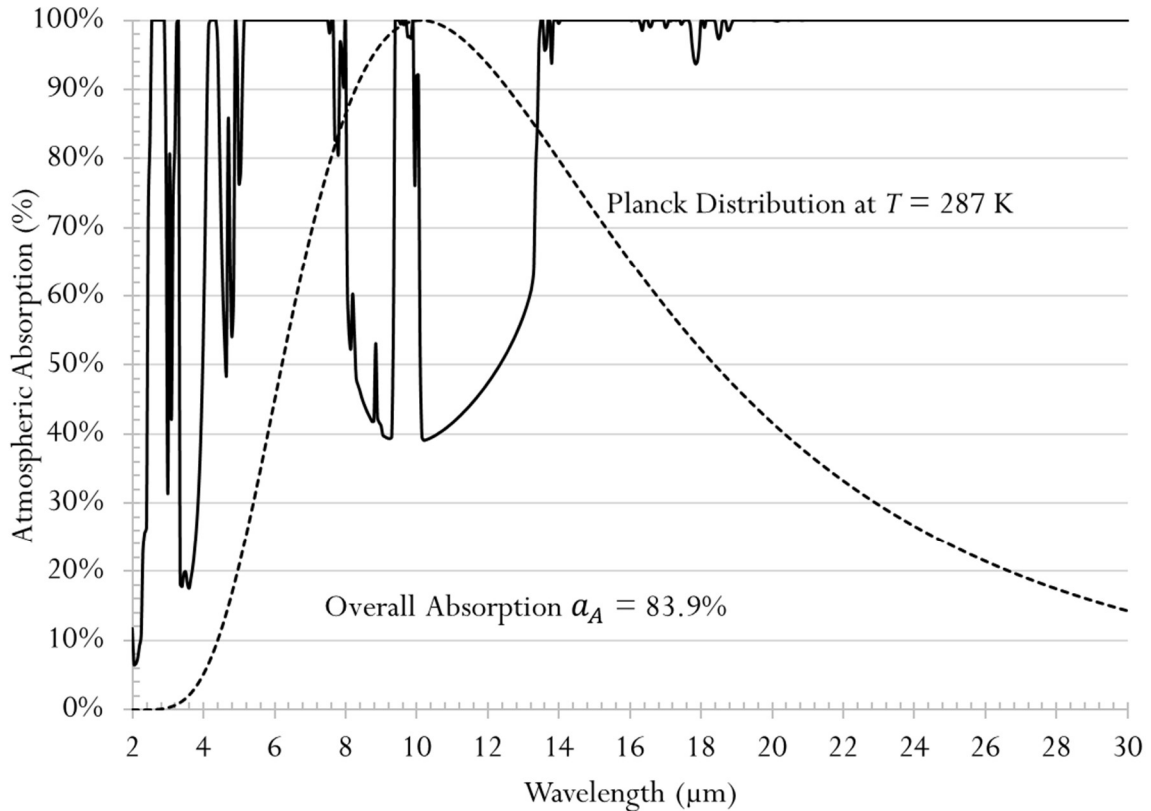
Figure 2: Absorption spectrum of the atmosphere above a surface at 287 K and composed carbon dioxide (280 ppm), methane (2 ppm), and ozone (1 ppm). The overall absorption by the atmosphere of a Planck distribution of emitted radiation at 287 K is 83.9%.

Figures 2 through 5 present the breakdown of the absorption spectrum in Figure 1 by atmospheric component analyzed. It is evident in Figure 3 for water vapor that water vapor is the dominant component in atmospheric absorption at 287 K.

Figure 4 for carbon dioxide captures a significant portion of radiation not captured by water vapor at 287K in the range of ~13 to 16 μm. The carbon dioxide absorption band at 4 μm produces minimal absorption due to the minimal amount of energy in the Planck distribution at 287 K and 4 μm.

Ozone in Figure 5 shows a significant absorption spectrum between 9 and 10 μm. Methane in Figure 6 absorbs predominantly between 7 and 8 μm but this radiation is absorbed as well by water vapor at 287 K. This suggests that the relative absorptivity of each component is dependent on the water vapor concentration. The water vapor concentration is estimated by the vapor pressure of water vapor at the surface that is dependent on surface temperature. Figures 7 and 8 show the overall absorption spectrum for a surface temperature of 273 K, and for solely water vapor above a surface at 273 K. Carbon dioxide, methane, and ozone spectrums remain the same in the model irrespective of surface temperature.
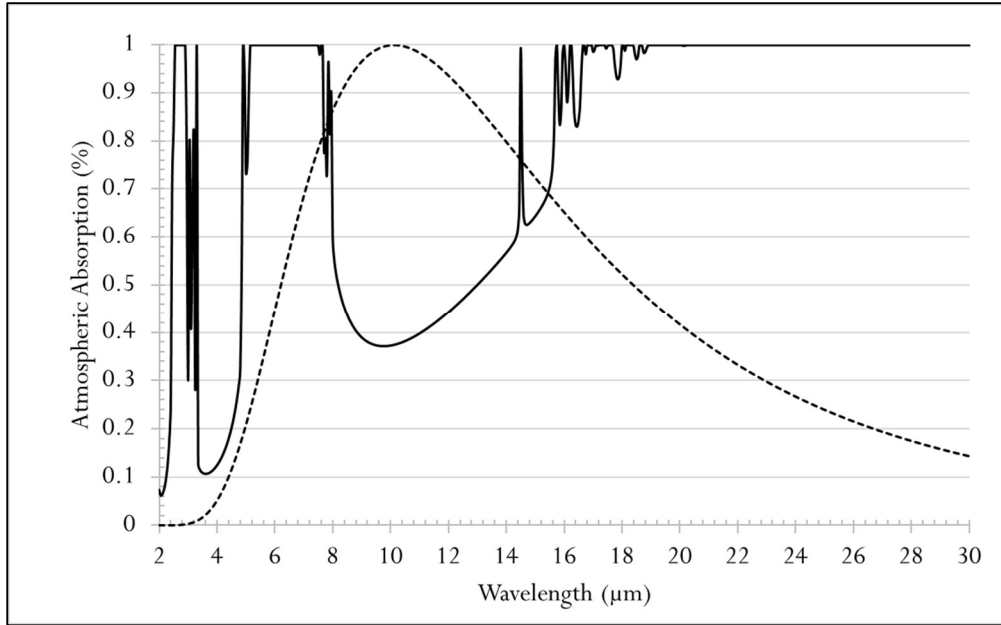
Figure 3: Absorption spectrum of water vapor at 287 K surface temperature (15,700 ppm) with the Planck distribution for 287 K (dashed line)
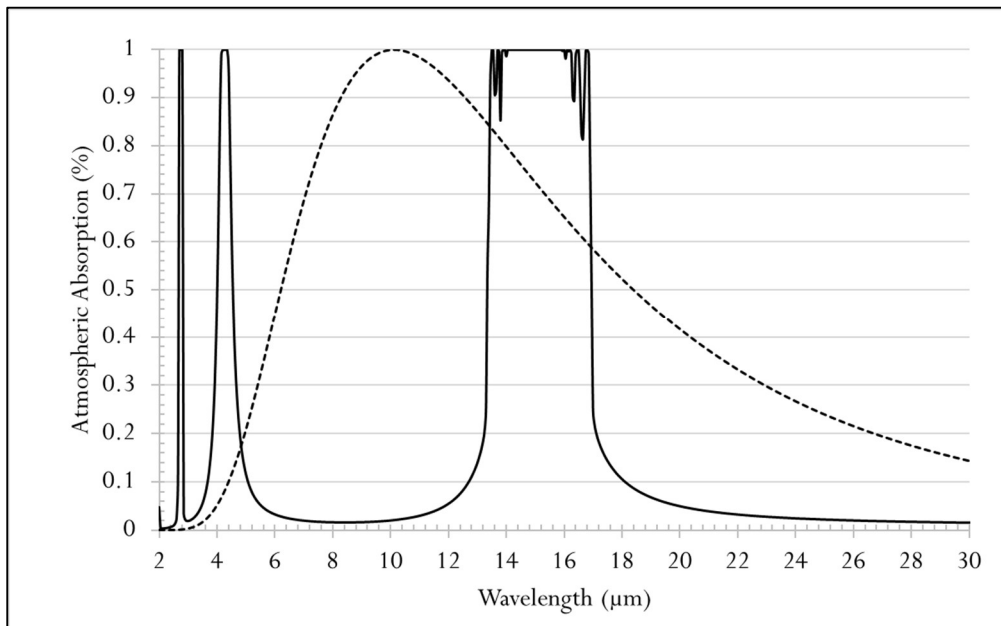

Figure 4: Absorption spectrum of carbon dioxide at 280 ppm with the Planck distribution for 287 K (dashed line)
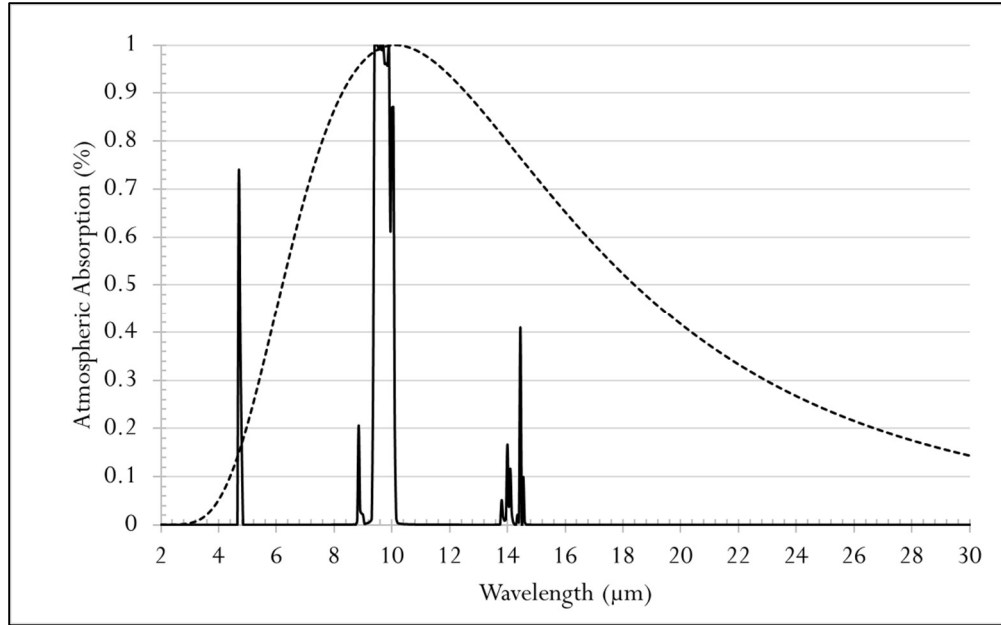
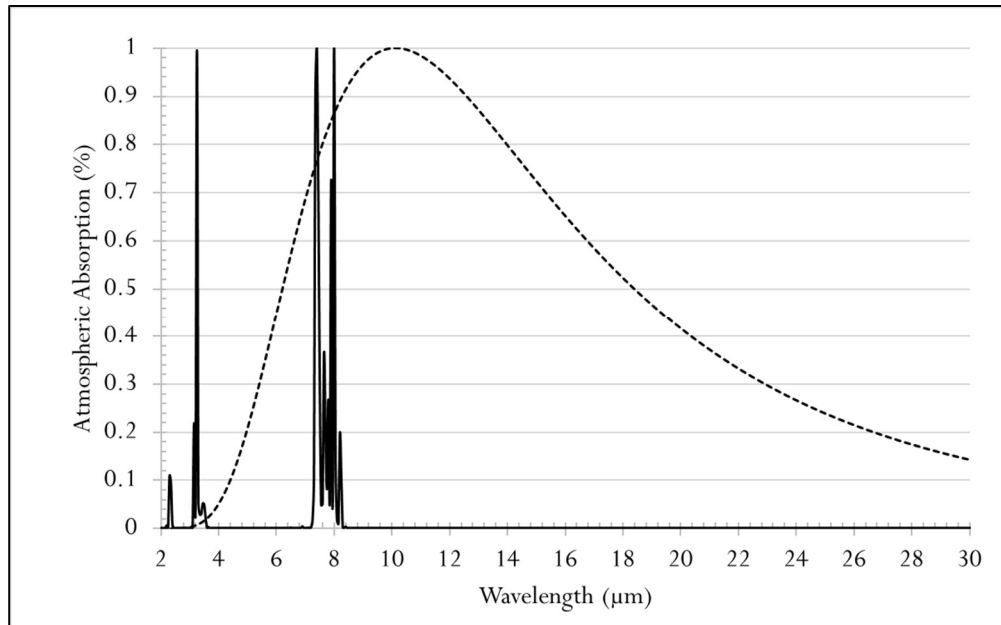Figure 5: Absorption spectrum of ozone at 1 ppm with the Planck distribution for 287 K (dashed line)

Figure 6: Absorption spectrum of methane at 2 ppm with the Planck distribution for 287 K (dashed line)

The effect of methane on the overall absorption is dependent on water vapor. The concentration of water vapor is dependent on temperature. The absorption spectrum of water vapor in Figure 3 extends nearly to 8 μm at 287 K while the absorption spectrum of water vapor in Figure 8 extends to slightly less than 8 μm. Cutting methane concentration to 1 ppm from 2 ppm reduces the overall absorption at 287 K surface temperature by 0.06% while doubling to 4 ppm increases overall absorption by 0.10%. The same analysis at 273 K surface temperature reduces the overall absorption by 0.12% for 1 ppm and increases by 0.18% for 4 ppm.

The pre-industrial (pre-1800) concentration of methane is estimated at 0.75 ppm while the modern concentration (2020) is approximately 2 ppm. The overall absorption increases from 83.86% to 83.94% using the model at 287 K surface temperature and from 77.18% to 77.34% at 273 K. The deduction is there is an effect on overall heat absorption by the atmosphere with changes in methane concentration, but the effect is minimal. In addition, this univariable analysis does not include the effects of methane

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concentration on atmospheric solar absorption, atmospheric heat emission, ozone depletion, and other parameters that cool or heat the surface of the Earth.
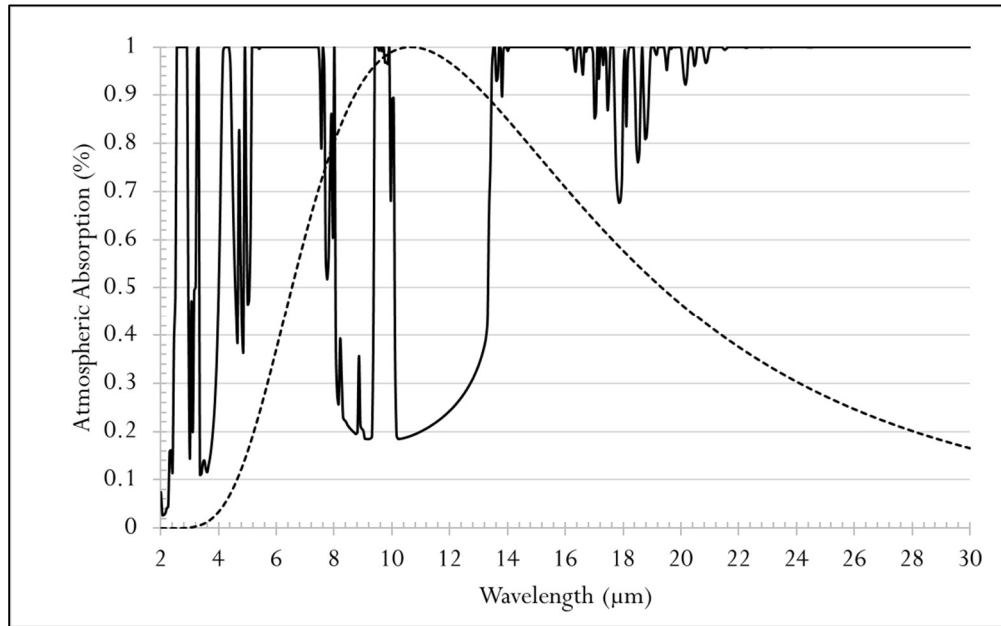

Figure 7: Absorption spectrum of the atmosphere above a surface at 273 K and composed of carbon dioxide (280 ppm), methane (2 ppm), and ozone (1 ppm). The overall absorption by the atmosphere of a Planck distribution of emitted radiation at 273 K is 77.3%.
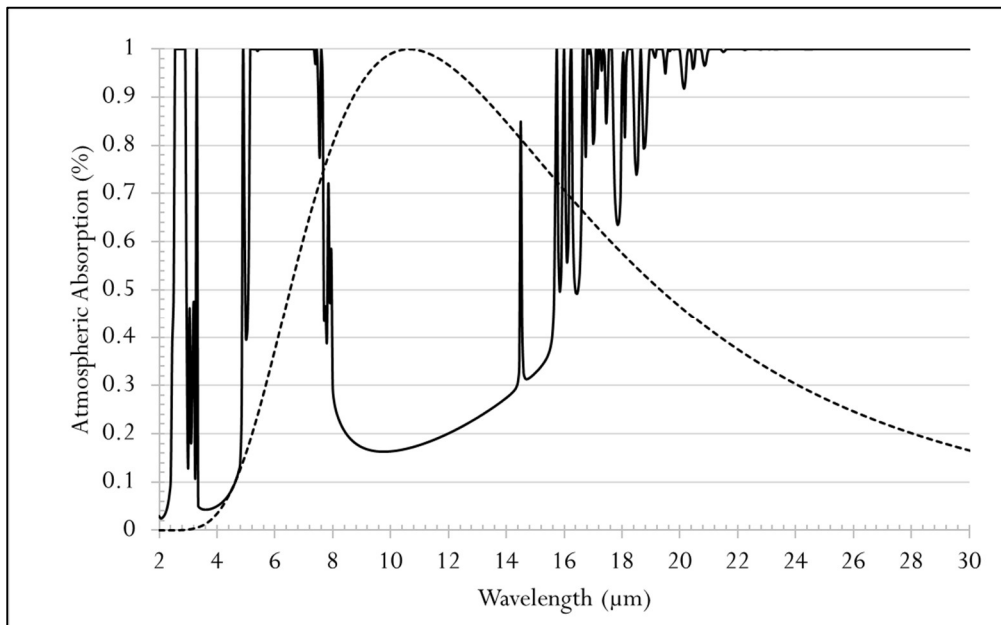

Figure 8: Absorption spectrum of water vapor at 273 K surface temperature (6,000 ppm) with the Planck distribution for 273 K (dashed line)

The overall absorption by the atmosphere of a Planck distribution of emitted radiation at the surface temperature is provided in Figure 9. Methane and ozone are maintained constant at 2 and 1 ppm, respectively, while carbon dioxide is varied from 0 to 1,000 ppm. It is evident that the overall absorption by carbon dioxide decreases with increasing surface temperature. This is

captured in Figures 2 and 7. As the surface temperature increases and the water vapor concentration increases, the water vapor absorption spectrum encroaches on the carbon dioxide absorption spectrum.
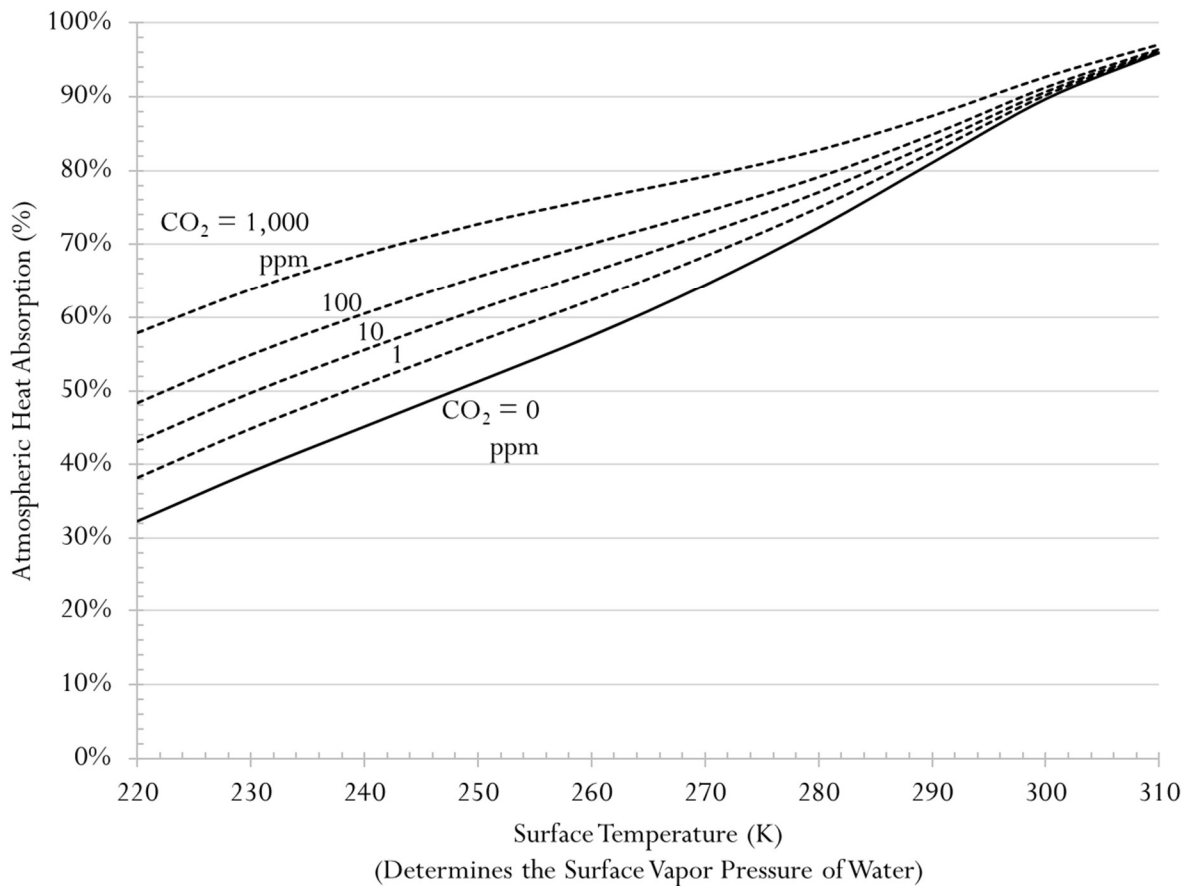

Figure 9: Overall absorption by the atmosphere with 1 ppm ozone and 2 ppm methane with varying surface temperature and logarithmically varying carbon dioxide concentration.

When applied to the Earth, polar surface temperatures typically range from 220 K to 280 K while equatorial surface temperatures typically range from 280 to 310 K. This suggests that the effect of carbon dioxide concentration in the atmosphere has a more pronounced effect in the polar regions than in the equatorial regions. For reference, the pre-industrial carbon dioxide concentration was ~280 ppm, the current concentration is ~420 ppm, and the lowest concentration during the last ice age was ~180 ppm. This range of concentrations suggest that the change in atmospheric heat absorption due to changes in carbon dioxide concentration is <1% overall for the entire Earth.

Atmospheric Heat Emission Versus Absorption

Infrared radiation is emitted by the atmosphere. Standard practice equates the heat emission to the heat absorption but this is incorrect. Atmospheric heat absorption is the absorption of surface sourced radiation by absorbing gases in the atmosphere. Atmospheric heat emission is the emission of radiation by atmospheric gases at the temperature of the atmosphere. Earth's average surface temperature is ~287 K while Earth's average atmospheric temperature is ~265 K (reference Woods paper). The difference in the Planck distribution of radiation at ~287 K and ~265 K is the source of the difference between atmospheric heat absorption and emission.

Applying the surface temperature Planck distributions to the absorption spectrum in Figure 2 produces an atmospheric heat absorption value of 83.9%. Applying the 265 K atmospheric temperature Planck distribution to the absorption spectrum in Figure 2 produces an atmospheric heat emission value of 85.3%. The reason for the higher value of emission compared to the absorption

is as the atmospheric temperature decreases away from the surface temperature, the Planck distribution for emission moves further into the region beyond 13-14 μm in Figure 2 with nearly 100% absorption. For temperatures experienced on Earth, the value of atmospheric heat emission is slightly higher than atmospheric heat absorption since the average atmospheric temperature is lower than the surface temperature of the Earth.

Conclusion

The atmospheric heat absorption spectrum requires solution of non-linear equations using significant computation power. This article developed an analytical equation for modeling the absorption of radiation for a given wavenumber as the radiation traverses the atmosphere. The model integrates an estimate of the variation in water vapor concentration with altitude and the diffuse nature of heat emission from a surface. The absorption spectrum was developed in Microsoft Excel™ using absorption line data from HITRAN (hitran.org) by only including absorption lines producing significant absorption of radiation.

Applying the absorption spectrum to a Planck distribution of heat energy from the surface deduced a profile for the overall atmospheric heat absorption at differing surface temperatures. The variation of the profile with variable carbon dioxide and methane concentrations were analyzed. The conclusion was increases in carbon dioxide and methane increase atmospheric heat absorption and atmospheric heat emission. The net effect on Earth's surface temperature and atmospheric temperature were not analyzed.

Finally, a method for calculating the overall atmospheric heat emission was developed. The conclusion was the heat emission requires knowledge of an atmospheric temperature for calculation. Since the atmospheric temperature was unknown, the overall atmospheric heat emission at differing surface temperatures was not computed. A general conclusion is since the atmospheric temperature is lower than the surface temperature of the Earth, the atmospheric heat emission is higher than the atmospheric heat absorption. This will be the focus of future efforts integrating an Earth temperature model with the analytical absorption model developed in this article.

Acknowledgements

Appendices

There are no appendices for this research.

References

[1] F. P. Incropera and D. P. DeWitt, Fundamentals of Heat and Mass Transfer, Hoboken: John Wiley & Sons, Inc., 2002.

[2] K. Woods, "Modeling Climatic Temperature Using The Stability Solution," *PROSE Journal of Thermal Science,* no. 000001, 2023.

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