Climatic Thermal Model With Validation Against Empirical Data

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Introduction

There is a deficiency in analytical thermal modeling of the Earth's climate leading to erroneous claims in academic textbooks. This article expands and validates the stability solution model in Woods [1] using the atmospheric heat absorption model in Woods [2] and atmospheric solar absorption model in Woods [3]. The validation is accomplished through comparison with data from the Climatic Research Unit (CRU) from the University of East Anglia [4]. The expanded stability solution model compares well with the empirical temperature measurements in the CRU dataset for latitudes between 66°S to 66°N encompassing >90% of the Earth's surface area. The model is a steady state thermal equilibrium model and does not capture the high solar variability and thermal storage heat transfer in the polar regions.

The model is applied to modeling changes in Earth's annually averaged surface temperature. The conclusion is the effect of carbon dioxide increases from human influence is minimal and changes in surface emissivity and evaporation fraction are likely causes of temperature changes on Earth. Carbon dioxide increasing from pre-industrial levels (280 ppm) to current levels (420 ppm) yields an increase of 0.15°C in Earth's average surface temperature. The model requires a carbon dioxide concentration of 10,000 ppm to produce a 1°C increase in Earth's average surface temperature.

Expanded Stability Solution Model

The climate model utilized is an expanded version of the stability solution model derived in Woods [1]. The modifications are an additional term in the surface energy balance, separation of evaporation and convection into two equations, addition of an atmospheric heat absorption equation, an atmospheric solar absorption equation, and addition of the average annual solar radiation impinging on a flat surface without the atmosphere and clouds.

The first equation is an energy balance on the surface. In Woods [1], the surface reflected solar energy was assumed to pass by the clouds back into space. This is not realistic because a portion is reflected by the bottom of the clouds back at the surface, and another portion is absorbed by the clouds. However, it is assumed that no cloud absorption occurs and 100% of the solar radiation reflected by the bottom of the clouds back towards the surface is absorbed by the surface. The modification from Woods [1] is the $(1 - x_c + \tau_c x_c)$ factor multiplied by the surface reflectivity (R_s).

$$\sigma\varepsilon_s T_s^4 + q_s = \frac{S}{4}(1 - R_r - \alpha_S)(1 - x_c + \tau_c x_c)\big(1 - R_s(1 - x_c + \tau_c x_c)\big) + (1 - x_c)\sigma\varepsilon_a T_a^4 + x_c\sigma\varepsilon_b T_b^4 \tag{1}$$

The second equation is an energy balance on the atmosphere including clouds. Woods [1] separates the atmosphere into a section covered by clouds without atmospheric heat absorption and a section without clouds with atmospheric absorption. The fraction of the atmosphere covered is denoted by the cloud fraction (x_c).

$$2(1 - x_c)\sigma\varepsilon_a T_a^4 + x_c\sigma\varepsilon_b T_b^4 + x_c\sigma\varepsilon_t T_t^4 = \frac{S}{4}\big((1 - x_c + \tau_c x_c)\alpha_S + x_c\alpha_c\big) + (\alpha_a(1 - x_c) + x_c)\sigma\varepsilon_s T_s^4 + q_s \tag{2}$$

The third and fourth equations are the evaporation and convection portions of the evaporation and convection equation in Woods [1]. The two equations are slightly modified to include the fraction of Earth with evaporation (F_w) multiplied by the driving convective force given by the condensable fraction of the atmosphere ($\Delta\rho$). The modification is made because as $F_w \to 0$, the convective equation in Eq. (4) should approach zero as well due to the lack of condensation driving convective flow. However, in Woods [1] this did not occur. Therefore, not only does F_w affect evaporation surface area but also the driving force for convection. For this verification and validation article, $F_w = 1$ is assumed for all latitudes.

$$q_s = 0.15F_w\left(\frac{P\Delta h D_m}{RT_a}\right)\left(\frac{g}{\nu D_m}\frac{F_w\Delta\rho}{\rho_o}\right)^{1/3}\left(\rho_w(T_s) - \rho_w(T_t)\right) \tag{3}$$

$$q_s = 0.15k\left(\frac{g}{\nu D}\frac{F_w\Delta\rho}{\rho_o}\right)^{1/3}(T_s - T_a) \tag{4}$$

The fifth equation is the first of the two stability solution equations modified with the change in Eq. (1).

$$\sigma\varepsilon_a T_a^4 - \sigma\varepsilon_b T_b^4 = \frac{S}{4}(1 - R_r - \alpha_S)(\tau_c - 1)\left(1 - 2R_s(1 - x_c + \tau_c x_c)\right) \tag{5}$$

The sixth equation is the second of the two stability solution equations. No modification is made to this equation as it is derived from Eq. (2).

$$\sigma\varepsilon_b T_b^4 + \sigma\varepsilon_t T_t^4 - 2\sigma\varepsilon_a T_a^4 - \sigma\varepsilon_s(1 - \alpha_a)T_s^4 = \frac{S}{4}(\alpha_c + (\tau_c - 1)\alpha_S) \tag{6}$$

The seventh equation is a linear approximation of atmospheric heat absorption derived from the model in Woods [2]. The limits are a maximum surface temperature of 320 K and a minimum carbon dioxide concentration (C_{CO2}) of 1 ppm for the approximation. The model results are shown in Figure 1.

$$\alpha_a = \begin{cases} 1 - \dfrac{320K - T_s}{100 + 50\log_{10}(C_{CO2})}, & 100ppm < C_{CO2} \\ 1 - \dfrac{320K - T_s}{150 + 25\log_{10}(C_{CO2})}, & 10ppm < C_{CO2} < 100\,ppm \\ 1 - \dfrac{320K - T_s}{160 + 15\log_{10}(C_{CO2})}, & 1ppm < C_{CO2} < 10\,ppm \end{cases} \tag{7}$$

The approximate model for atmospheric heat absorption is logarithmic with carbon dioxide concentration and linear with temperature.

The eighth equation is a correlation of solar absorption data from Woods [3] for atmospheric solar absorption and varies with surface temperature and latitude (l).

$$\alpha_S = 0.125 + 0.015\left(1 + erf\left(\frac{l - 60°}{15°}\right)\right) + \left(0.0017 + 0.00045\left(\frac{l}{90°}\right)\right)(T_s - 260K) \tag{8}$$

The ninth equation provides the latitudinal variation of average annual solar radiation impinging on a flat surface without the atmosphere or cloud interactions. The equation is a correlation based on a solar radiation analysis integrating the declination angle with time, day number, latitude, and hour angle. The hourly impinging solar radiation is averaged over the entire year leading to Eq. (9).

$$\frac{S}{4} = 295.2 + 120.7\cos(2l) + 8.7\sin(2l) \tag{9}$$

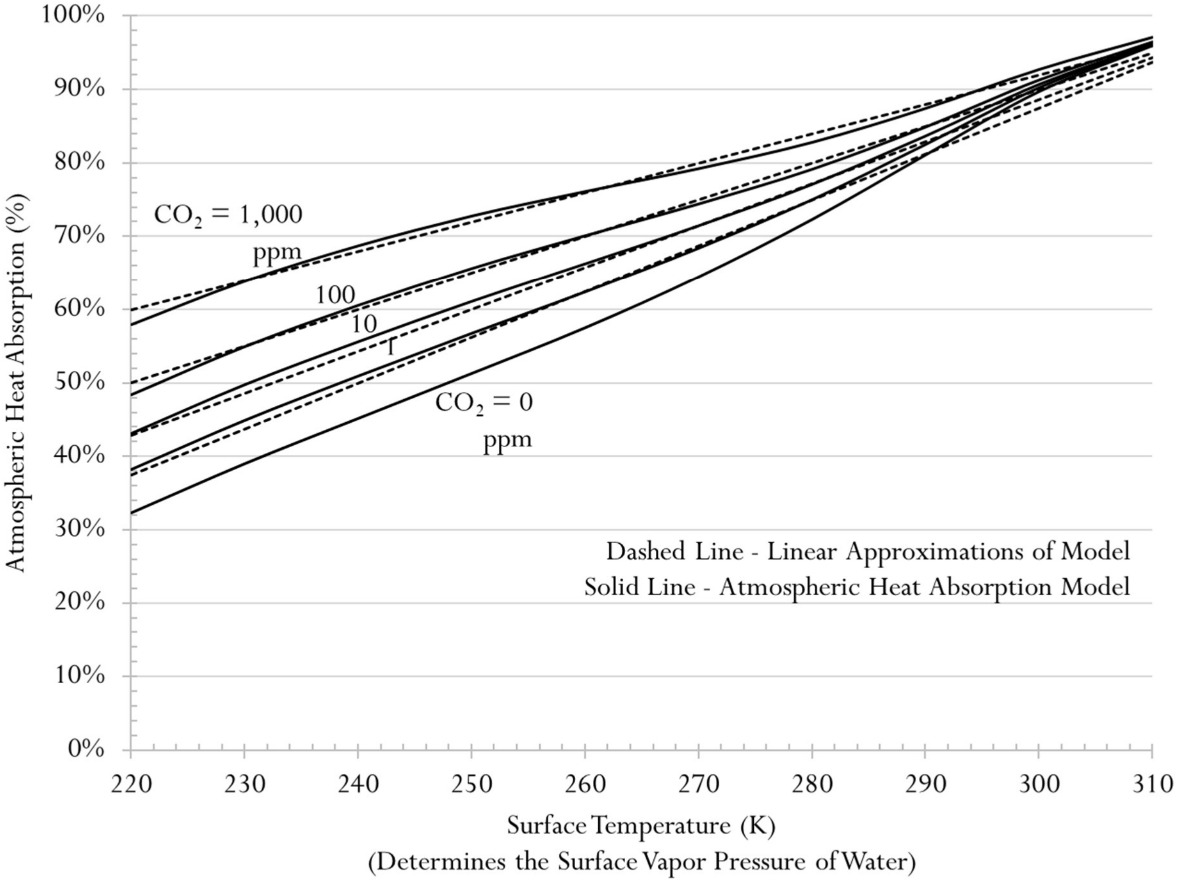

Figure 1: Atmsopheric heat absorption linear approximation (dashed line) versus model (solid line) from Woods [2]

The average annual solar radiation varies from 416 W/m^2 at the equator down to 174.5 W/m^2 at the poles. This variation is the driving force for the temperature variation across the Earth. The final two equations are the atmospheric heat absorption equals the atmospheric heat emission ($\alpha_a = \varepsilon_a$) and the cloud fraction is assumed $x_c = 0.67$ (67%). The value of $S/4$ for each latitude is shown in Figure 2. In addition, the maximum average daily solar radiation is shown for each latitude. Note the maximum average daily solar radiation is near the average annual solar radiation at the equator while the maximum average daily solar radiation deviates significantly from the average annual solar radiation. At the poles, the maximum average daily solar radiation is 543 W/m^2 while the minimum is 0 W/m^2. At the equator, the maximum is 432 W/m^2 while the minimum is 396 W/m^2. Since the climate model uses the annually average solar radiation, the results are more accurate near the equator than the poles.

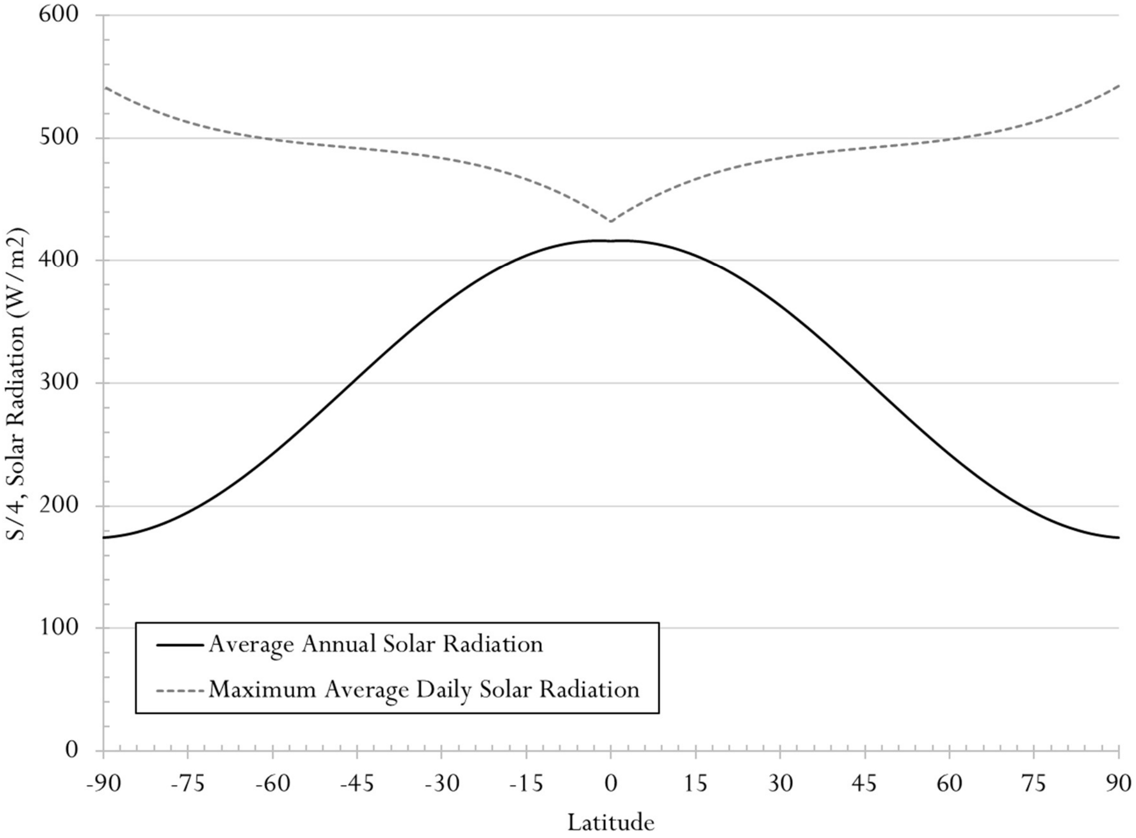

Figure 2: Annual average solar radiation (solid line) and maximum average daily solar radiation (dashed line) versus latitude

Model Results

Applying the climate model using the parameter values in Table 1 produces the profiles shown in Figure 3. The temperatures are the annually averaged temperature for the given latitude. Important to note is the cloud bottom temperature exceeds the surface temperature in the equatorial region. A hypothetical reason for this inversion is the latent heat of condensation of water vapor leads to heating of the cloud droplets on the bottom side of the clouds to a temperature above the surface.

Table 1: Equation parameter values utilized in Figure 3

Parameter	Value	Unit
Cloud Fraction	0.67	-
Surface Solar Reflectivity	0.10	-
Surface Emissivity	0.95	-
Rayleigh Scattering	0.05	-
Cloud Top Emissivity	1.00	-
Cloud Bottom Emissivity	1.00	-
Surface Evaporation Fraction	1.00	-
Carbon Dioxide Concentration	280	ppm
Cloud Transmissivity	0.55	-
Cloud Absorptivity	0.10	-
Cloud Reflectivity	0.35	-

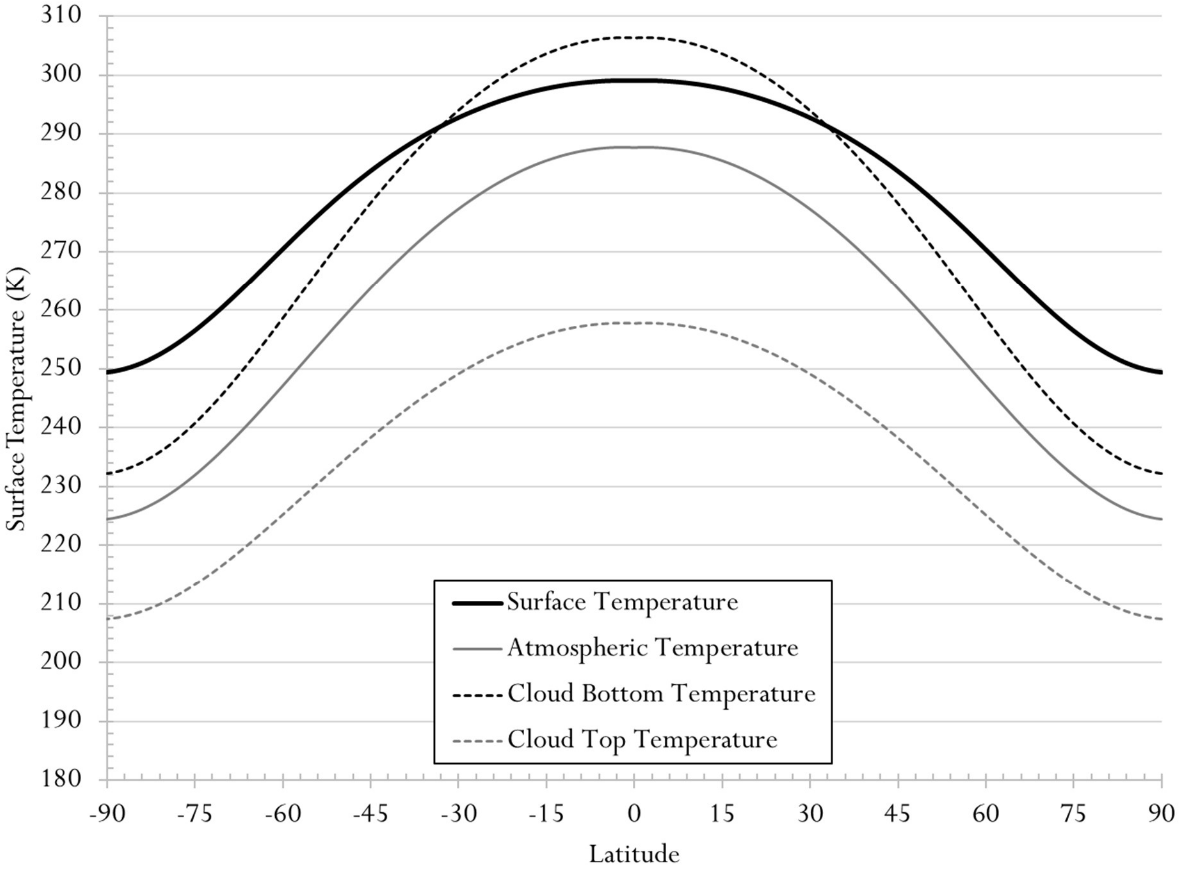

Figure 3: Expanded stability solution climate model output of annually average temperatures versus latitude

Model Validation

The Climatic Research Unit (CRU) of the University of East Anglia maintains empirical data from surface temperature measurements that are consolidated into average values versus latitude. The CRU average surface temperature data profile is presented in Figure 4 against the climate model average annual surface temperature profile. The expanded stability solution model agrees well with the CRU data for latitudes from 60°S to 60°N and there is a significant difference between the north and south polar temperatures.

Utilizing the maximum average daily solar radiation for the poles of 543 W/m^2, the model surface temperature is 309.5 K (36.4°C) while the minimum average daily solar radiation (0 W/m^2) yields a temperature of 0 K (-273.1°C), a difference of 309.5 K. This range of equilibrium temperatures relies heavily on the thermal storage capacity of the ground and atmosphere yielding a dynamic heat transfer problem that is not the focus of this article. Also, for comparison, the equatorial maximum and minimum average daily solar radiation values yield a temperature difference of 3.4°C. The minimum average daily solar radiation of 0 W/m^2 occurs for all latitudes from the Arctic circle to North pole and from the Antarctic circle to the South pole. Without thermal storage the polar temperatures would drop to 0 K (-273.1°C) during the winter. It is not a surprise that the polar regions differ significantly from the model as the average solar radiation varies drastically throughout the year.

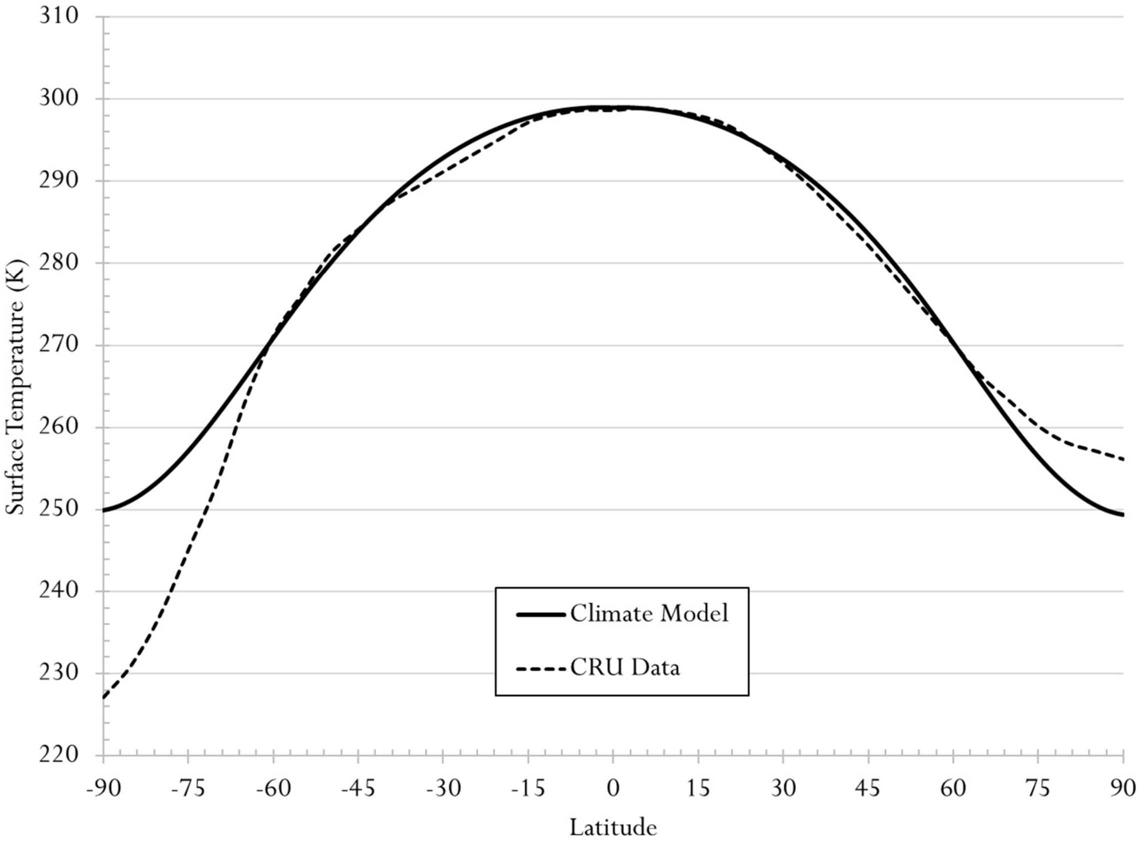

Figure 4: Climate model surface temperature profile versus latitude compared to CRU empirical data surface temperature profile versus latitude

However, just to touch on the differences, the north pole is in the Arctic Ocean while the south pole is on land in Antarctica. The Arctic Ocean experiences freezing and thawing of sea ice leveraging the 334 kJ/kg latent heat of fusion of water to produce heat during the winter. This heat is located at the solid-liquid interface of the freezing water and ice, and is conducted through the ice to the surface where it is a main component in maintaining the surface temperature above 200 K during winter with no solar radiation. Air and water convection are the other components that provide heat to the Arctic region during winter.

Antarctica experiences slight freezing and thawing of surface snow and ice but is heavily reliant on the heat capacity of snow and ice to capture heat during the summer and release the heat during the winter. In addition, the Antarctic is influenced by the elevation change from sea level to an average elevation of 2,500 m (8,200 ft) over the continent with a maximum elevation of 4,100 m (13,450 ft). The adiabatic lapse rate in the atmosphere is ~10 K/km causing a temperature range of ~41 K due to elevation changes. Modifying the average calculated surface temperature using a simple linear assumption of elevation increase to the average elevation as you approach the south pole (90° S) produces the Antarctica temperature profile in Figure ?. At 90° S, the calculated surface temperature is about 250 K and the modified surface temperature is 250 K – 25 K = 225 K where 25 K is 10 K/km times the 2.5 km average elevation. The modification compares well with the CRU data suggesting that the deviation in the model from the CRU data is predominantly due to elevation change for Antarctica.

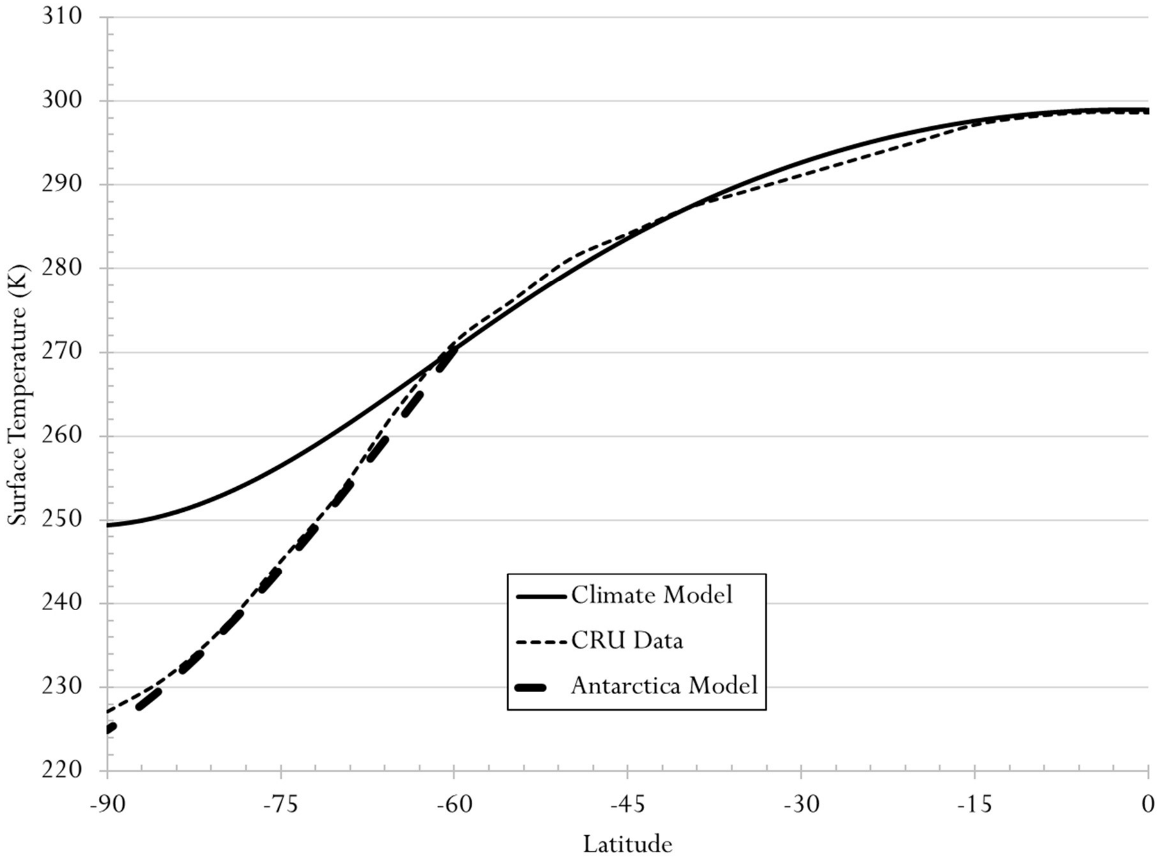

Figure 5: Antarctica modified surface temperature model compared to CRU data and climate model.

Other Validation Considerations

The overall albedo of the Earth is 30% but claimed to range from 28% to 31% depending on the source [5, 6, 7, 8]. Calculating the albedo requires an understanding of how much atmospheric absorption occurs before solar radiation impinges on the clouds. Since this analysis does not delve into this value, two equations are utilized to approximate the overall albedo. The first includes atmospheric solar absorption in the calculation of cloud reflection (Eq. 10) and the second does not include atmospheric solar absorption (Eq. 11):

$$R = R_r + (1 - R_r - \alpha_S)(1 - x_c + \tau_c x_c)^2 R_s + (1 - R_r - \alpha_S) x_c R_c \tag{10}$$

$$R = R_r + (1 - R_r - \alpha_S)(1 - x_c + \tau_c x_c)^2 R_s + (1 - R_r) x_c R_c \tag{11}$$

The calculated values are R = 26.7% and 31.0%, respectively.

The Earth thermal heat flux emitted is ~235 W/m² [9]. The calculated value in Eq. (23) is the addition of surface sourced emission, atmospheric sourced emission, plus cloud sourced emission.

$$q_E = \sigma \varepsilon_s {T_s}^4 (1 - \alpha_a)(1 - x_c) + \sigma \varepsilon_a {T_a}^4 (1 - x_c) + x_c \sigma \varepsilon_c {T_c}^4 \tag{12}$$

Here the model calculated heat emission is q_E = 243.8 W/m². Using this calculated heat emission produces an overall albedo of R = 28.6%.

Model Application – Human Caused Temperature Change

The effect of human sourced carbon dioxide (CO_2) emission on the temperature of the Earth is of interest in society. Over the past 150 years, the atmospheric concentration of CO_2 has increased from the pre-industrial equilibrium concentration of about 280 ppm to the current concentration of 420 ppm. This increase is attributed to an increase in the surface temperature of the Earth due to the carbon dioxide increasing atmospheric heat absorption. These concentrations are applied in the expanded stability solution model without varying the other parameters in Table 1 and the results are shown in Figure 6.

Increasing the CO_2 concentration from 280 ppm to 420 ppm increases the annually averaged surface temperature by 0.15°C over the entire Earth. The temperature increase at the equator is 0.06°C while at the poles the increase is 0.46°C. The reasoning behind this difference is shown in Figure 1 where the influence of carbon dioxide on atmospheric heat absorption is lower at warmer surface temperatures compared to cooler surface temperatures. The required CO_2 concentration to cause a 1°C increase in the annual average surface temperature is 10,000 ppm.

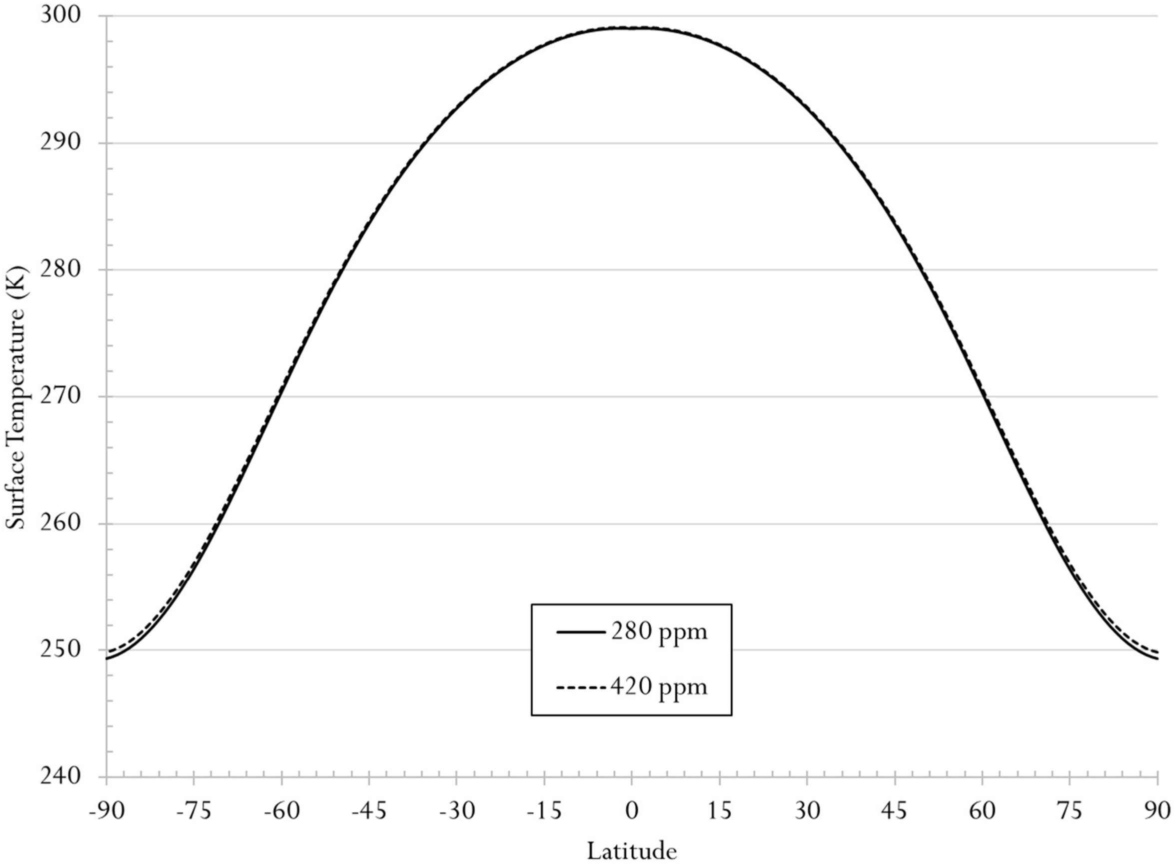

Figure 6: Model Temperature profiles varying carbon dioxide concentration from the pre-industrial concentration of 280 ppm (solid line) to the current concentration of 420 ppm (dashed line)

Woods [1] evaluates the relative influence of the parameters in Table 1 and suggests that the surface reflectivity is key parameter affecting Earth's surface temperature. However, with the integration of re-reflection of solar radiation by the bottom of clouds in the expanded stability solution model, changes in the surface reflectivity produce a negligible change in Earth's surface temperature.

Aside from atmospheric heat absorption from CO_2, the key parameters affecting Earth's surface temperature are Earth's surface emissivity and surface evaporation fraction. Figure 7 shows the surface emissivity and surface evaporation fraction that produce a 1°C increase in Earth's average annual surface temperature along with the 0.15°C increase from the 420 ppm carbon dioxide concentration.

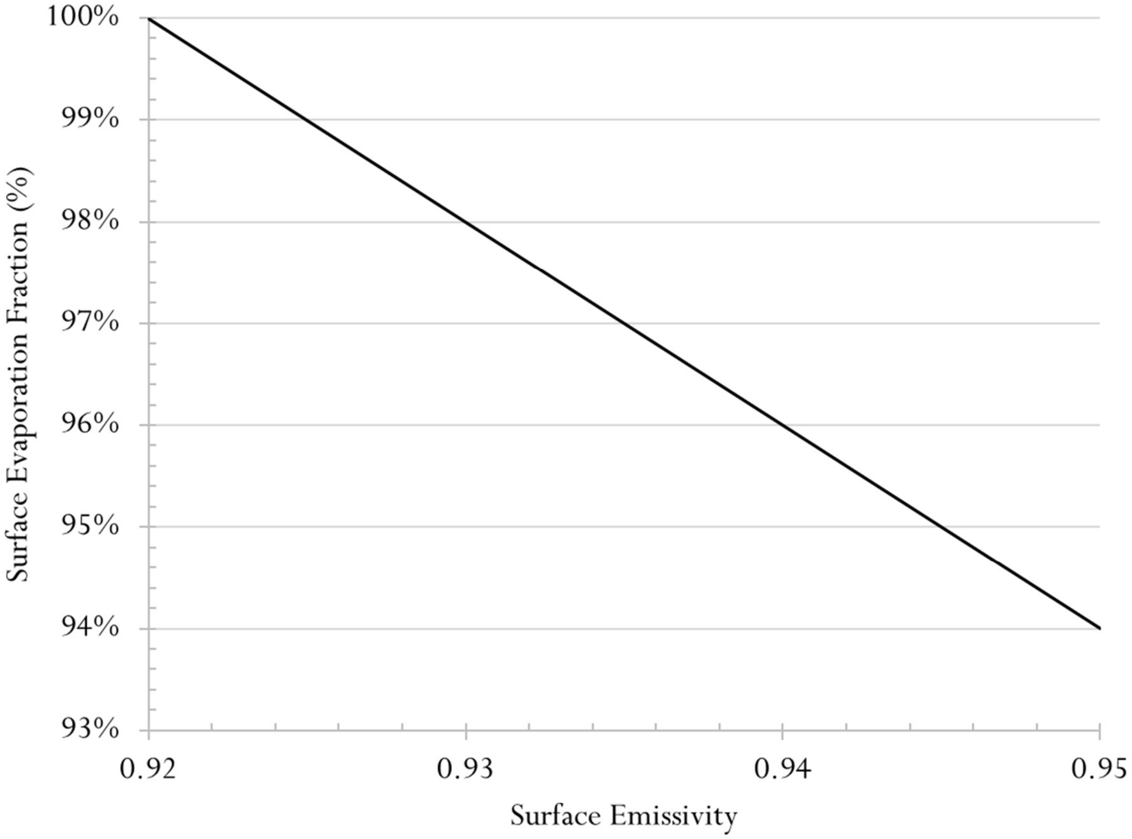

Figure 7: Average surface emissivity and evaporation fraction required to yield a 1°C increase in Earth's average annual surface temperature when added to the 0.15°C increase caused by CO_2 increasing to 420 ppm from 280 ppm

Earth's surface is ~30% land and ~70% ocean. Human agriculture and urbanization encompass about 1/3 of Earth's land or ~10% of Earth's surface. A surface without plants will dry quicker than a surface with plants as the roots of the plants access water deeper in the soil and utilize this water for transpiration. A dry surface possesses a lower emissivity than a wet surface. For example, the emissivity of dry sand is 0.68 while the emissivity of water is 0.96, from Incropera and Dewitt [10]. It is likely that the effect of human agriculture and urbanization has led to a lower average surface emissivity and lower surface evaporation fraction causing an increase in Earth's average annual surface temperature beyond the 0.15°C increase by carbon dioxide.

Woods [2] evaluates the absorption of heat by methane in the atmosphere and deduces that the effect of increasing methane concentrations from human influences from 0.75 ppm to 2 ppm to be an increase of <0.1% to the atmospheric heat absorption parameter. The effect of methane is minimal due to overlap of methane absorption bands with water vapor absorption bands especially in equatorial regions. In polar regions, the effect of lower temperatures shifts the Planck distribution of heat emission away from the methane absorption bands between 2 and 8.3 μm. As a result, the effect of methane increasing from 0.75 ppm to 2 ppm on climatic temperatures yields an increase in average annual surface temperature of <0.03°C globally.

Conclusion

The standard models and deductions in academic textbooks, articles, and on websites about Earth's thermal climatic balance are incorrect. This article produced an analytical model for analyzing thermal variations of the Earth's climate. The model applies nine equations to calculate Earth's average surface temperature, atmospheric temperature, cloud temperatures, atmospheric heat absorption, atmospheric solar absorption, solar radiation, and evaporation and convection heat flux from the surface to the atmosphere for a given latitude. The model is solved in standard academic software such as Microsoft Excel™. The model showed good agreement when compared to empirical data for the latitudinal surface temperature profile from the climate research unit [4] providing validation of the model.

Applying the model to evaluate the effect of human emitted carbon dioxide and methane on Earth's annually averaged surface temperature produced an increase of 0.15°C and <0.03°C, respectively. It is not likely that human caused changes in carbon dioxide and methane concentration are contributing a significant amount in Earth's average annual surface temperature changes. Rather, surface emissivity and evaporation fraction are the strongest influence on Earth's average annual surface temperature, and human agriculture and urbanization are a possible cause of changes in surface emissivity and evaporation fraction.

Acknowledgements

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Appendices

There are no appendices for this research.

References

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