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Modeling Climatic Temperature Using The Stability Solution

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Introduction

There is a deficiency in analytical thermal modeling of the Earth's climate leading to erroneous claims in academic textbooks. The standard claims are, first, heat absorbing gases in the atmosphere increase the Earth's surface temperature by about 30°C and this increase is called the "Greenhouse Effect" and, second, carbon dioxide is the dominant factor affecting the "Greenhouse Effect". For example, Chang [1], a standard textbook in Chemistry, claims on p. 713, "If the atmosphere did not contain carbon dioxide, Earth would be 30°C cooler." Cengel and Boles [2] on p. 22 state, "The greenhouse effect makes life on earth possible by keeping the earth warm (about 30°C warmer)." In the book *Physics of Climate*, Peixoto [3] claims on p.118, "the greenhouse effect due to the existence of the atmosphere is $\Delta T = 33$ K." These quotes are non-exhaustive as these claims are stated in numerous university textbooks, articles, and on numerous government and organizational websites [4, 5, 6, 7]. These claims are sourced from erroneous deductions from the Earth blackbody radiator model.

The Earth blackbody radiator model calculates the emission temperature of the Earth using an energy balance equating absorbed solar radiation to heat emitted by the Earth shown in Eq. (1). The system boundary is above the atmosphere limiting the heat transfer to radiation heat transfer. The percent of solar energy reflected by the Earth back into space, known as the mean Earth albedo (R), is about 30% [3].

$$T_E = \left(\frac{S(1-R)}{4\sigma}\right)^{1/4} \tag{1}$$

Here S is the solar constant at 1365 W/m^2 incident on the cross-sectional area of the Earth, and σ is the Stefan-Boltzmann constant for heat emission over the surface area of the Earth's outer atmosphere. The '4' in the denominator is from the ratio of cross-sectional area to surface area of the spherical system boundary around Earth. The emission temperature of the Earth $T_E =$ 255 K is a combination of surface, atmospheric, and cloud emission temperatures.

The erroneous claims are sourced from a comparison of the Earth emission temperature (T_E) to the actual average surface temperature (T_s) of ~288 K producing $\Delta T = T_s - T_E =$ ~33 K. The standard deduction is this ΔT difference is due to heat absorption by certain gases in the atmosphere (e.g. water vapor, carbon dioxide, ozone) termed the "Greenhouse effect." [1, 2, 3] The error is the assumption that T_E is Earth's *surface* temperature without atmospheric heat absorption (Greenhouse Effect) when it is a combination of surface, atmosphere, and cloud emission temperatures. The second erroneous claim is the differential is solely or predominantly due to carbon dioxide. It is evident in Incropera and Dewitt [8] on p. 823-826 that this claim is invalid because the heat emissivity/absorptivity of water vapor alone is about 80% of that produced by water vapor plus carbon dioxide at atmospheric concentrations and 300 K.

A proper analysis begins with an Earth surface with either no atmosphere or a perfectly transparent atmosphere, no evaporation or convection, and surface albedo rather than the Earth's albedo. The majority of the Earth's albedo is cloud and atmospheric reflection [9]; meaning the surface albedo $R_s < R$. Assuming Earth's surface is a perfect blackbody with no atmosphere, then the surface albedo $R_s = 0$, and $T_s = 278.5$ K. Nevers [10] (p.442) includes the perfect blackbody analysis but regresses to the blackbody radiator analysis.

The logical next step is to assume the Earth surface as a gray surface with the properties of water since the Earth is predominantly oceans. The heat emissivity $\varepsilon_s = 0.96 \pm 0.02$ [8] and surface solar albedo $R_s = 0.10 \pm 0.05$. Eq. (1) becomes Eq. (2) with surface emissivity added in the denominator and Earth albedo replaced with surface solar albedo.

$$T_s = \left(\frac{S(1 - R_s)}{4\sigma\varepsilon_s}\right)^{1/4} \tag{2}$$

Here T_s = 274.1 K. If a "Greenhouse Effect" is to be claimed, a more appropriate temperature increase is ΔT_{new} = ~14 K. Nevertheless, the gray surface model is deficient as it neglects atmosphere, clouds, evaporation, and convection. This article integrates these components into the model.

Beyond the blackbody, blackbody radiator, and gray surface models, Jacob [11] (p.128) discusses a simple greenhouse model using two energy balance equations. The first energy balance is applied to the Earth where solar energy absorbed is emitted by a combination of surface and atmosphere heat emission in Eq. (3). The second equation is an energy balance on the atmosphere equating surface emitted heat absorbed by the atmosphere to the heat emitted by the atmosphere in Eq. (4). The fraction of the surface emission that reaches the system boundary above the atmosphere is f approximately equal to the atmospheric heat absorptivity (α_a) and atmospheric heat emissivity (ε_a).

$$\frac{S(1 - R)}{4} = (1 - f)\sigma T_s^4 + f\sigma T_a^4 \tag{3}$$

$$2f\sigma T_a^4 = f\sigma T_s^4 \tag{4}$$

The average surface and atmospheric temperatures are solved producing Eq. (5) and Eq. (6).

$$T_s = \left(\frac{S(1 - R)}{4\sigma\left(1 - \frac{f}{2}\right)}\right)^{1/4} \tag{5}$$

$$T_a = \left(\frac{T_s^4}{2}\right)^{1/4} \tag{6}$$

The surface emissivity is neglected, evaporation and convection from the surface to the atmosphere are neglected, and the albedo in Eq. (3) and Eq. (5) includes the reflection from clouds which are neglected in the rest of the problem. Jacob [11] concludes that if T_s = 288 K is assumed, then f = 0.77 and T_a = 241 K.

Jacob [11] then transitions into a discussion of 'radiative forcing' where parameters, such as a change in atmospheric composition, are integrated into the problem and the change in surface temperature is evaluated and quantified as 'forcing' on the climate. The change in surface temperature is also converted into a 'global warming potential' parameter evaluating the relative influence of different atmospheric constituents on the surface temperature relative to carbon dioxide. Although more intricate than the blackbody radiator model, Jacob's [11] two equation model is incorrect making the 'radiative forcing' and 'global warming potential' deductions inaccurate.

This article develops an analytical thermal model calculating the average surface, atmospheric, cloud bottom, and cloud top temperatures, as well as the average evaporation and convection from the surface to the atmosphere. The model separates the problem into an energy balance on the surface and an energy balance on an atmosphere including clouds. Rayleigh back-scattering, atmospheric absorption of solar energy, cloud absorption of solar energy, surface albedo, atmospheric heat absorption/emission, surface heat absorption/emission, surface evaporation, atmospheric condensation, and surface to atmosphere convection are all integrated into the model.

The analysis begins by restating the two-equation model integrating surface albedo, surface emissivity/absorptivity, atmospheric solar absorptivity, atmospheric heat absorptivity/emissivity, and Rayleigh back-scattering (atmospheric albedo) yielding an increase in the average surface temperature calculation from the graybody model of T_s = 274.1 K to T_s = 305.1 K. The model then integrates evaporation and convection through inclusion of a third equation utilizing standard free convection relations for heat and mass transfer from a horizontal heated surface [8] yielding an average surface temperature T_s = 287.6 K.

Finally, a stability assumption is made based on empirical cloud fraction data enabling expansion of the three-equation model into a five-equation model integrating clouds and yielding an average surface temperature of T_s = 286.4 K. The five-equation model is called the Analytical Stability Solution Earth Thermal Model.

The article concludes with a justification of the parameter values utilized in the article, an evaluation of the relative influence of each parameter on the model, an uncertainty analysis of the temperature and heat flux values calculated, and a discussion of the relative influence of water vapor and carbon dioxide on the Earth's surface temperature.

Analysis

The problem is separated into an energy balance on the Earth's surface and an energy balance on the Earth's atmosphere. The surface energy balance system boundary is between the Earth's surface and the first atmospheric molecule above the surface. The atmospheric energy balance system ranges from the surface energy balance boundary up to space but neglects clouds. Eq. (3) and (4) are the energy balances on Earth's surface and atmosphere, respectively.

The left sides of both equations are the emission by the surface and atmosphere, respectively. The right side of Eq. (7) is the solar radiation reaching Earth's surface plus the heat emitted by the atmosphere and incident on the surface. The solar radiation reaching the surface is reduced by Rayleigh back-scattering into space (R_r) and atmospheric solar absorption (α_S). The portion of the solar radiation reaching the surface that is absorbed is $1 - R_s$. The right side of Eq. (8) is the amount of sunlight absorbed by the atmosphere plus the surface emitted heat absorbed by the atmosphere.

$$\sigma\varepsilon_s T_s^4 A_s = S(1 - R_r - \alpha_S)(1 - R_s)A_c + \sigma\varepsilon_a T_a^4 A_s \tag{7}$$

$$2\sigma\varepsilon_a T_a^4 A_s = S\alpha_S A_c + \alpha_a\sigma\varepsilon_s T_s^4 A_s \tag{8}$$

Here ε_a is the atmospheric heat emissivity assumed equal to the atmospheric heat absorptivity (α_a). A_s and A_c are the surface and cross-sectional areas of the Earth, respectively, while T_s and T_a are the mean surface and atmospheric temperatures, respectively. Inserting Eq. (8) into Eq. (7) and solving for the mean surface temperature produces Eq. (9).

$$T_s = \left(\frac{\frac{S}{4}(1 - R_r - \alpha_S)(1 - R_s) + \frac{S}{8}\alpha_S}{\sigma\varepsilon_s\left(1 - \frac{\alpha_a}{2}\right)}\right)^{1/4} \tag{9}$$

Solving Eq. (8) for atmospheric temperature produces Eq. (10).

$$T_a = \left(\frac{\varepsilon_s T_s^4}{2} + \frac{S\alpha_S}{8\sigma\varepsilon_a}\right)^{1/4} \tag{10}$$

The surface reflection varies based on surface characteristics, incident angle of solar radiation, and other factors that change with time as the Earth rotates and orbits around the sun. About 70% of the Earth's surface is water. Water is highly transparent to light normal to its surface with reflectivity of about 0.03 becoming highly reflective at high angles of incidence (>60°). The remaining 30% of Earth's surface is land composed of vegetation, dirt, sand, and rock assumed to have a similar emissivity to water but a higher reflectivity of 0.25 [8]. The overall surface reflectivity is assumed $R_s = 0.10 \pm 0.05$, and the surface emissivity is assumed that of water $\varepsilon_s = 0.96 \pm 0.02$ [8]. The atmospheric absorptivity and emissivity are assumed $\varepsilon_\mathrm{a} = \alpha_\mathrm{a} = 0.85 \pm 0.05$ [8]. Rayleigh back-scattering is assumed $R_r = 0.05 \pm 0.03$ and atmospheric solar absorptivity is assumed $\alpha_\mathrm{S} = 0.15 \pm 0.05$.

The two-equation model average surface and atmosphere temperatures are T_s = 305.1 K and T_a = 261.7 K, respectively. This exceeds the actual mean surface temperature of the Earth of T_s = 288 K (14.9°C). The increase from the graybody model surface temperature T_s = 274.1 K to the two-equation model surface temperature is +31 K. The two-equation model is expanded in the next section to include a third equation quantifying the heat flux from the Earth's surface to the atmosphere driven by evaporation and convection. It is a fallacy to believe that the atmospheric heat absorption causes a +31 K increase because atmospheric absorption is predominantly water vapor which condenses and evaporates transferring heat as shown in the next section.

Three-Equation Model: Evaporation and Convection

Water evaporates from the Earth's surface and condenses in the atmosphere releasing heat and producing a pressure gradient in the atmosphere that drives convective air flow. Water evaporates cooling the surface and condenses in the atmosphere releasing heat. The average annual rainfall over the Earth is approximately 1 meter equal to a heat flux due to evaporation of 80 W/m^2 [12]. Since the average solar flux on Earth is $S/4 =$ 341 W/m^2, the 80 W/m^2 evaporative heat flux is significant.

Adding an assumed average surface evaporation and convection flux (q_s) to Eqs. (7) and (8) produces Eqs. (11) and (12) for the Earth surface and atmospheric energy balances.

$$\sigma\varepsilon_s T_s^4 A_s + q_s A_s = S(1 - R_r - \alpha_S)(1 - R_s)A_c + \sigma\varepsilon_a T_a^4 A_s \tag{11}$$

$$2\sigma\varepsilon_a T_a^4 A_s = S\alpha_S A_c + \alpha_a \sigma\varepsilon_s T_s^4 A_s + q_s A_s \tag{12}$$

Solving for surface and atmospheric temperature yields Eqs. (13) and (14), respectively.

$$T_s = \left(\frac{\frac{S}{4}(1 - R_r - \alpha_S)(1 - R_s) + \frac{S}{8}\alpha_S - \frac{q_s}{2}}{\sigma\varepsilon_s\left(1 - \frac{\alpha_a}{2}\right)} \right)^{1/4} \tag{13}$$

$$T_a = \left(\frac{\varepsilon_s T_s^4}{2} + \frac{\frac{S}{4}\alpha_S + q_s}{2\sigma\varepsilon_a} \right)^{1/4} \tag{14}$$

The value of q_s is quantified using heat and mass transfer theory for free convection. Water evaporating into air flow increases the relative air pressure at the surface while condensing water vapor generates a decrease in relative air pressure in the upper troposphere. This pressure differential ($\Delta\rho$) is related to the mole fraction of water vapor in the atmosphere (equal to partial pressure) and quantified by the temperature dependent vapor pressure of water divided by the overall air pressure.

The Grashof number (Gr) is of the order 10^{17} to 10^{21} for atmospheric convection cells. The mean Nusselt number (Nu) follows the general form $\overline{Nu} = C \cdot (GrPr)^n$ and the mean Sherwood number is $\overline{Sh} = C \cdot (GrSc)^n$. The parameters $C = 0.15$ and $n = 1/3$ apply for the upper side of a heated surface [8]. It should be noted that in [8] the form of this equation is only valid for Rayleigh numbers, $Ra = GrPr < 10^{11}$ however this relation is utilized for Ra of about 10^{17} to 10^{21} nonetheless. The general form of the Nusselt and Sherwood numbers are

$$\overline{Nu} = 0.15\left(\frac{gL^3}{\nu D}\frac{\Delta\rho}{\rho_o}\right)^{1/3} \qquad \overline{Sh} = 0.15\left(\frac{P}{RT_a}\right)\left(\frac{gL^3}{\nu D_m}\frac{\Delta\rho}{\rho_o}\right)^{1/3} \tag{15}$$

Here g is the gravitational constant (9.8 m/s^2), ν is the kinematic viscosity of air, D is the thermal diffusivity, and D_m is the mass diffusivity of water in air. The parameter $\rho_o \approx 1 - \Delta\rho/2$. An advantageous effect of $n = 1/3$ is the heat and mass convection coefficients have no characteristic length dependency as the characteristic length term (L) cancels. The total evaporation and convection rate from the surface is shown in Eq. (15). This is the third equation in the Three-Equation Model.

$$q_s = 0.15k\left(\frac{g}{\nu D}\frac{\Delta\rho}{\rho_o}\right)^{1/3}(T_s - T_a) + 0.15\left(\frac{P\Delta h {D_m}^{2/3}}{RT_a}\right)\left(\frac{g}{\nu}\frac{\Delta\rho}{\rho_o}\right)^{1/3} F_w\Delta\rho \tag{16}$$

Here $\Delta\rho = \rho_w(T_s) - \rho_w(T_a)$ where $\rho_w(T_s)$ is the mole fraction of water vapor just above Earth's surface determined using the vapor pressure of water at Earth's surface temperature, $\rho_w(T_a)$ is the atmospheric mole fraction of water determined using the vapor pressure of water at the atmospheric temperature, k is the thermal conductivity of air, Δh is the latent heat of evaporation of water (~45 kJ/mol), F_w is the fraction of the Earth's surface area that is wet, and P is the surface pressure applied to determine the surface molar gas density using the ideal gas law. Dry desert covers approximately 10% of the Earth's surface with near zero evaporation leading to $F_w \approx 0.9$. However, since the equator evaporates significantly more water than the temperate or polar regions and most deserts are in the equatorial regions, the effect on evaporation is higher leading to $F_w \approx 0.8 \pm 0.1$.

Solving for the mean surface temperature, atmospheric temperature, and evaporative and convective flux with $F_w = 0.8$ produces $T_s = 287.6$ K, $T_a = 265.9$ K, $q_s = 114.5$ W/m^2. The calculated average surface temperature is approximately the measured average surface temperature. The next section expands the Three-Equation Model to a Five-Equation Model integrating clouds.

Five-Equation Model: Analytical Stability Solution Earth Thermal Model

Clouds are three dimensional amorphous structures reflecting, emitting, scattering, transmitting, and absorbing both heat and solar radiation. Modeling clouds is dubious but a simple assumption is that clouds are a two-dimensional surface between the ground and the system boundary that does not interact with the atmospheric heat emissions and covers a fraction of the surface of the Earth characterized by the cloud fraction (x_c). The cloud fraction throughout the year is relatively stable at $x_c = 0.67 \pm 0.02$ [13] as shown from NASA Earth Observation (NEO) data from 2019 presented in Figure 1.

The surface energy balance includes heat emitted by the fraction of sky covered by clouds and heat emitted by the atmosphere in the fraction of the sky not covered by clouds. The atmospheric energy balance includes heat emission by the atmosphere, top of clouds, and bottom of the clouds. This generates 5 unknowns requiring 5 equations to solve. The first two equations are modifications of the surface and atmosphere energy balances shown in Eqs. (17) and (18) and the third is a modification to the evaporation and convection heat flux.

The modification to the surface energy balance utilizes the cloud fraction to modify the sunlight reaching the surface and separate the heat incident on the surface into a fraction sourced from the atmosphere and a fraction sourced from the bottom of the clouds. The modification to the atmospheric energy balance similarly separates heat emission, and atmospheric solar and heat absorption using the cloud fraction. The clouds are assumed opaque to heat.

$$\sigma\varepsilon_s T_s^4 + q_s = \frac{S}{4}(1 - R_r - \alpha_S)(1 - x_c + \tau_c x_c)(1 - R_s) + (1 - x_c)\sigma\varepsilon_a T_a^4 + x_c\sigma\varepsilon_b T_b^4 \tag{17}$$

$$\begin{aligned} 2(1 - x_c)\sigma\varepsilon_a T_a^4 + x_c\sigma\varepsilon_b T_b^4 + x_c\sigma\varepsilon_t T_t^4 \\ = \frac{S}{4}\big((1 - x_c + \tau_c x_c)\alpha_S + x_c\alpha_c\big) + (\alpha_a(1 - x_c) + x_c)\sigma\varepsilon_s T_s^4 + q_s \end{aligned} \tag{18}$$

Here T_b and T_t are the cloud bottom and top temperatures, respectively, and $\varepsilon_b = \alpha_b = 0.975 \pm 0.025$ and $\varepsilon_t = \alpha_t = 0.975 \pm 0.025$ are the assumed cloud bottom and top heat emissivity and absorptivity, respectively. $\tau_c = 0.55$, $R_c = 0.35$, and $\alpha_c = 0.10$ are the assumed cloud solar transmissivity, reflectivity, and absorptivity, respectively [14].

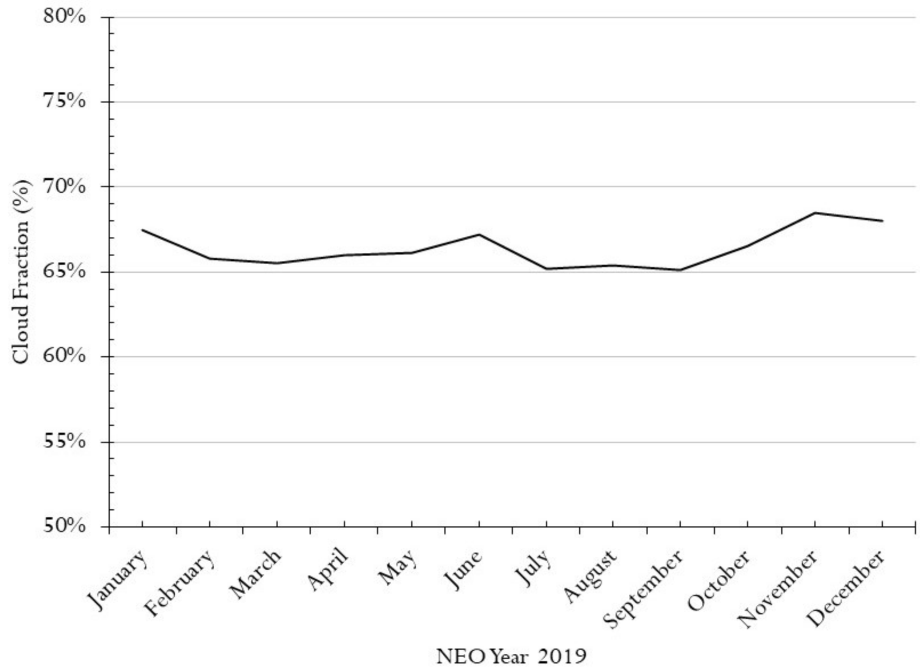

Figure 1

The modification to the equation for convection and evaporation is Eq. (19) with $\Delta\rho = \rho_w(T_s) - \rho_w(T_t)$ where the condensable gas partial pressure in the atmosphere is determined by the cloud top temperature rather than the atmospheric temperature. The cloud top temperature more accurately describes the coldest surface temperature of cloud water droplets.

$$q_s = 0.15k\left(\frac{g}{\nu D}\frac{\Delta\rho}{\rho_o}\right)^{1/3}(T_s - T_a) + 0.15F_w\left(\frac{P\Delta h {D_m}^{2/3}}{RT_a}\right)\left(\frac{g}{\nu}\frac{\Delta\rho}{\rho_o}\right)^{1/3}\left(\rho_w(T_s) - \rho_w(T_t)\right) \tag{19}$$

The additional two equations are determined from the stability solution.

Stability Solution

The stable cloud fraction throughout the year suggests that the Earth is in an equilibrium. The Stability Solution assumes that the equilibrium is due to all temperatures and heat fluxes at a minimum at x_c meaning the derivative of all temperatures and heat fluxes with respect to x_c equal zero at x_c. The final two equations in the Five-Equation Model are obtained by differentiating Eqs. (17) and (18) with respect to x_c and setting the temperature and heat flux derivates equal to zero, shown in Eqs. (20) and (21).

$$\sigma\varepsilon_a T_a^4 - \sigma\varepsilon_b T_b^4 = \frac{S}{4}(1 - R_r - \alpha_S)(1 - R_s)(\tau_c - 1) \tag{20}$$

$$\sigma\varepsilon_b T_b^4 + \sigma\varepsilon_t T_t^4 - 2\sigma\varepsilon_a T_a^4 - \sigma\varepsilon_s(1 - \alpha_a)T_s^4 = \frac{S}{4}(\alpha_c + (\tau_c - 1)\alpha_S) \tag{21}$$

The system of equations is solved using an iterative method where, first, the temperatures are calculated through matrix multiplication with an assumed evaporation and convection heat flux. Second, the evaporation and convection equation are solved using the calculated temperatures. Finally, these two calculations are iterated until convergence. The Five-Equation Model yields a mean surface temperature T_s = 286.4 K, a mean atmospheric temperature T_a = 266.2 K, a mean cloud bottom temperature T_b = 282.6 K, a mean cloud top temperature T_t = 244.5 K, and a mean evaporation and convection rate q_s = 121.3 W/m^2.

Parameters

The structure of the model was the focus of this article. Below are short justifications for the parameter values utilized and estimates of the uncertainties in the parameter values. First principle derivations of more accurate parameter values will be the focus of future work.

Atmospheric Heat Absorption and Emission

The atmospheric heat absorption and emission parameter is calculated in Incropera and Dewitt [8] through a combination of the emissivity of water vapor and carbon dioxide. Utilizing the graphs from p. 823-826 in [8], the gas emissivity is the addition of the emissivity of water vapor equal to ~0.7 at 300 K plus the emissivity of carbon dioxide equal to ~0.2 at 300 K minus a correction factor equal to ~0.05 at 300 K; totaling ~0.85. The uncertainty of this value is assumed ± 0.05 yielding an atmospheric heat emissivity of $\varepsilon_a = 0.85 \pm 0.05$.

It is assumed that the atmospheric heat emissivity and absorptivity are equal, however, the absorptivity and emissivity are slightly different due to the incident radiation absorbed or emitted. For example, the Planck distribution emitted by the Earth's surface and absorbed by the atmosphere is characterized by the average surface temperature but the Planck distribution emitted by the atmosphere is characterized by the average atmospheric temperature. This slight difference leads to a difference of ~0.01 in the atmospheric heat absorption and atmospheric heat emission.

Rayleigh Scattering

Rayleigh scattering is a mechanism that scatters low wavelength solar radiation through elastic interactions with atmospheric molecules producing the blue color of the sky during the day. The scattering causes a portion of solar radiation to reflect across the atmospheric energy balance system boundary decreasing the amount of sunlight reaching the surface and absorbed in the atmosphere. Rayleigh scattering is inversely proportional to the fourth power of the wavelength. Approximately 25% of blue light at 400 nm is scattered. Utilizing a Planck distribution based on the sun's surface temperature of 5800 K, the approximate total Rayleigh scattering is about 13%. However, a portion of the radiation up to 415 nm is absorbed by ozone and oxygen. Therefore, about 10% is Rayleigh scattered with 50% back-scattered suggesting that reflection back into space $R_r = 0.05 \pm 0.01$.

Atmospheric Solar Absorptivity

The atmospheric solar absorptivity is the amount of sunlight that is absorbed by gases in the atmosphere in transit to the surface. Ozone and oxygen absorb a large portion of sunlight up to 415 nm in the stratosphere totaling about α_{Ozone} = 3-4%. Water vapor absorbs solar radiation in the 718 nm, 810 nm, 935 nm, 1130 nm, 1380 nm, 1880 nm, and 2680 nm bands. Assuming each absorption band is ± 25 nm and the solar radiation is a Planck distribution based on the sun's surface temperature of 5800 K, a coarse estimate of the total atmospheric solar absorption is $\alpha_S = 0.15 \pm 0.05$.

Considerations and Conclusion

Parameter Variation

The Five-Equation Model parameters are varied to determine their relative influence on the average surface temperature of the Earth. Each parameter is varied to produce an increase of 1°C in the average surface temperature of the Earth. The results are shown in Table 1. These parameters are varied independently but a change in temperature may be due to a combination of parameters varying to a lesser extent. For example, human agriculture has expanded over the past two centuries to cover about 10% of the Earth's surface. Implementing agriculture on land previously covered by trees or other vegetation directly influences the surface emissivity, surface reflectivity, and the wet surface fraction. A decrease in the surface emissivity to 0.95, surface solar reflection to 0.09, and wet surface fraction to 75% yields a 1.0°C increase in mean surface temperature. In addition, the model does not integrate the possibility of one parameter affecting another. For example, a change in ozone absorption in the atmospheric solar absorption will affect the Rayleigh back-scattering value.

Table 1

Parameter	Initial Parameter Value	Value Yielding 1°C Increase in Surface Temperature
Surface Emissivity	0.96	0.928
Surface Solar Reflection	0.10	0.08
Earth Wet Surface Fraction	80%	60%
Atmos Heat Absorption	0.850	0.885
Solar Constant (W/m^2)	1365	1393
Rayleigh Backscattering	0.050	0.031
Cloud Top Emissivity	1.00	0.68
Atmos Solar Absorption	0.15	0.08

Uncertainty Analysis

Table 2 shows the uncertainty assumed for each parameter in the Five-Equation Model. The assumed value is constant irrespective of latitude. Applying an uncertainty calculation assuming each parameter is independent produces an average surface temperature parameter uncertainty of ±3.7 K.

Table 2

Parameter	Initial Parameter Value	Temperature Uncertainty (°C)	Assumed Uncertainty of Value (±)
Surface Emissivity	0.96	0.60	0.02
Surface Solar Reflection	0.10	2.30	0.05
Earth Wet Surface Fraction	80%	0.10	10%
Solar Constant (W/m^2)	1365	1.10	10
Rayleigh Backscattering	0.05	1.10	0.01
Cloud Top Emissivity	0.975	0.60	0.025
Cloud Bottom Emissivity	0.975	0.10	0.025
Atmos Solar Absorption	0.15	2.10	0.05
Atmos Heat Absorption/Emission	0.85	0.90	0.05

In addition to parameter uncertainties, there is the mathematical uncertainty associated with utilizing an average Earth surface temperature where the physics of the problem are non-linear. The surface temperature of the Earth varies from about 220 to 320 K depending on time and location. The standard deviation of the Earth's surface temperature averaged over the area of the Earth is about 13°C. For example, for T_s = 286.4 K, if half of the Earth's surface is T_s – 13°C and the other half is T_s + 13°C, and $0.5(T_s - 13)^4 + 0.5(T_s + 13)^4 = {T_{s,new}}^4$, then $T_{s,new}$ = 287.2 K. Therefore, a mathematical calculation uncertainty associated with determining the average surface temperature is estimated as $T_{s,new} - T_s$ = 0.8 K.

The overall uncertainty assumes that the mathematical uncertainty and parameter uncertainty are orthogonal yielding a total uncertainty $u_{T_s} = ((0.8\text{K})^2 + (3.7\text{K})^2)^{1/2} = \pm 3.8$ K. The Five-Equation Model average surface temperature is T_s = 286.4 K ± 3.8 K; encompassing the claimed average surface temperature of about 288 K from textbooks and other sources [3, 7, 10, 11].

Relative Influence of Carbon Dioxide and Water Vapor

When the influence of carbon dioxide is removed from the atmospheric heat emissivity and absorptivity value in the model, the new value estimate is $\alpha_a = \varepsilon_a = 0.7$ for water vapor only. Applying this to the model with all other parameters remaining the same produces T_s = 282.4 K, T_a = 264.1 K, T_b = 272.2 K, T_t = 244.5 K, and q_s = 92.4 W/m^2. Evaporation/condensation, heat absorption, and solar absorption solely from water and water vapor is estimated to increase Earth's surface temperature by 8.4°C. Earth's surface temperature calculated with carbon dioxide included is T_s = 286.4 K. Therefore, the heat absorption and emission from carbon dioxide is estimated to increase Earth's surface temperature by about 4.0°C. This is in stark contrast to the 30°C value claimed by Chang [1]. The carbon dioxide effect on emissivity is non-linear with concentration. A 50% reduction in carbon dioxide concentration reduces emissivity from ~0.85 to ~0.83 yielding a decrease in T_s by ~0.5 K using the Five-Equation Model.

Other Validation Considerations

The overall albedo of the Earth is 30% but claimed to range from 28% to 31% depending on the source [3, 7, 10, 11]. The calculated Earth albedo from the Five-Equation Model parameters is shown in Eq. (22) as the addition of the Rayleigh backscattering, surface reflection, and cloud reflected solar radiation traversing back across the atmospheric energy balance system boundary.

$$R = R_r + (1 - R_r - \alpha_S)(1 - x_c + \tau_c x_c)R_s + (1 - R_r - \alpha_{Ozone})x_c R_c \tag{22}$$

Here R = 32.2% ± 3.2% encompassing the claimed Earth albedos.

The Earth thermal heat flux emitted is ~235 W/m^2 [15]. The calculated value in Eq. (23) is the addition of surface sourced emission, atmospheric sourced emission, plus cloud sourced emission.

$$q_E = \sigma\varepsilon_s {T_s}^4(1 - \alpha_a)(1 - x_c) + \sigma\varepsilon_a {T_a}^4(1 - x_c) + x_c\sigma\varepsilon_c {T_c}^4 \tag{23}$$

Here q_E = 230.2 W/m^2 ± 16.4 W/m^2 encompassing the claimed Earth thermal heat flux.

Conclusion

The standard models and deductions in academic textbooks, articles, and on websites about Earth's thermal climatic balance are incorrect. The Analytical Stability Solution Earth Thermal Model is a viable alternative for analyzing thermal variations of the Earth's climate and is suggested as a replacement for university curricula. The model applies five equations to calculate Earth's average surface temperature, atmospheric temperature, cloud bottom surface temperature, cloud top surface temperature, and the evaporation and convection heat flux from the surface to the atmosphere. The model is solved in standard academic software including Microsoft Excel™, and MathCAD. Future research includes

- First principle derivation of parameters such as the atmospheric heat absorptivity/emissivity, atmospheric solar absorptivity/emissivity, and Rayleigh scattering.
- Integration of variable cloud properties based on droplet diameter Mie Scattering and radiation absorption theory.
- A latitudinal analysis comparing the model to empirically measured average surface temperature values with latitude.
- Evaluation of the influence of atmospheric components on temperature using the Stability Solution Model.

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Appendices

There are no appendices for this research.

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